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Cohesive fracture evaluation in brittle materials using the BEMCRACKER2D Software

Avaliação de fratura coesiva em materiais frágeis usando o software BEMCRACKER2D

Evaluación de fracturas cohesivas en materiales frágiles mediante el software BEMCRACKER2D

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ABSTRACT

Understanding and predicting the behavior of cracks in brittle materials, such as concrete, is essential to prevent structural failures and ensure the longevity and safety of engineering structures. This study utilizes Linear Elastic Fracture Mechanics (LEFM) and the Dual Boundary Element Method (DBEM) to evaluate the effects of cohesive fractures on crack opening displacement and crack growth in brittle materials, with a specific emphasis on concrete. The in-house developed BEMCRACKER2D program, coupled with the BEMLAB2D graphical interface, forms the core of the analysis methodology. The methodology involves a detailed assessment of cohesive fractures in concrete that are simulated using the fictitious crack model, where the fracture zone is represented by cohesive forces, modeled through both linear and bilinear curves acting on the crack surfaces. This approach allows for a nuanced simulation of the fracture process, capturing the complex interactions within the material as the crack propagates. To verify and validate the adopted methodology, two classic examples from the literature on brittle materials were carefully selected. These examples provide benchmark scenarios that are well-documented, allowing for a rigorous comparison of the simulation results with established data. The accuracy and reliability of the BEMCRACKER2D program were demonstrated through these examples, showing that the program can predict crack behavior with a high degree of precision. The findings highlight the potential of BEMCRACKER2D as a valuable tool for the analysis of cohesive fractures in brittle materials. Its ability to accurately model and predict crack growth and displacement not only reinforces its applicability in academic research but also suggests significant practical applications in engineering design and safety assessments.

Keywords: Cohesive Fracture. Plain Concrete. DBEM. BEMCRACKER2D

RESUMO

Compreender e prever o comportamento de fissuras em materiais frágeis, como o concreto, é essencial para evitar falhas estruturais e garantir a longevidade e a segurança das estruturas de engenharia. Este estudo utiliza a Mecânica da Fratura Elástica Linear (LEFM) e o Método do Elemento de Contorno Duplo (DBEM) para avaliar os efeitos das fraturas coesivas no deslocamento de abertura de trincas e no crescimento de trincas em materiais frágeis, com ênfase

específica no concreto. O programa BEMCRACKER2D desenvolvido internamente, juntamente com a interface gráfica BEMLAB2D, forma o núcleo da metodologia de análise. A metodologia envolve uma avaliação detalhada das fraturas coesivas no concreto que são simuladas usando o modelo de trinca fictícia, em que a zona de fratura é representada por forças coesivas, modeladas por meio de curvas lineares e bilineares que atuam nas superfícies da trinca. Essa abordagem permite uma simulação diferenciada do processo de fratura, capturando as interações complexas dentro do material à medida que a trinca se propaga. Para verificar e validar a metodologia adotada, dois exemplos clássicos da literatura sobre materiais frágeis foram cuidadosamente selecionados. Esses exemplos fornecem cenários de referência que são bem documentados, permitindo uma comparação rigorosa dos resultados da simulação com dados estabelecidos. A precisão e a confiabilidade do programa BEMCRACKER2D foram demonstradas por meio desses exemplos, mostrando que o programa pode prever o comportamento de trincas com um alto grau de precisão. As descobertas destacam o potencial do BEMCRACKER2D como uma ferramenta valiosa para a análise de fraturas coesivas em materiais frágeis. Sua capacidade de modelar e prever com precisão o crescimento e o deslocamento de trincas não apenas reforça sua aplicabilidade em pesquisas acadêmicas, mas também sugere aplicações práticas significativas em projetos de engenharia e avaliações de segurança.

Palavras-chave: Fratura coesiva. Concreto simples. DBEM. BEMCRACKER2D

RESUMEN

Comprender y predecir el comportamiento de las grietas en materiales frágiles, como el hormigón, es esencial para prevenir fallos estructurales y garantizar la longevidad y seguridad de las estructuras de ingeniería. Este estudio utiliza la Mecánica de Fractura Elástica Lineal (LEFM) y el Método de Elemento Límite Dual (DBEM) para evaluar los efectos de las fracturas cohesivas en el desplazamiento de apertura de grieta y el crecimiento de grieta en materiales frágiles, con un énfasis específico en el hormigón. El programa de desarrollo propio BEMCRACKER2D, junto con la interfaz gráfica BEMLAB2D, constituye el núcleo de la metodología de análisis. La metodología implica una evaluación detallada de las fracturas cohesivas en hormigón que se simulan utilizando el modelo de grieta ficticia, donde la zona de fractura está representada por fuerzas cohesivas, modeladas a través de curvas lineales y bilineales que actúan sobre las superficies de la grieta. Este enfoque permite una simulación matizada del proceso de fractura, capturando las complejas interacciones dentro del material a medida que se propaga la grieta. Para verificar y validar la metodología adoptada, se seleccionaron cuidadosamente dos ejemplos clásicos de la bibliografía sobre materiales frágiles. Estos ejemplos proporcionan escenarios de referencia que están bien documentados, lo que permite una comparación rigurosa de los resultados de la simulación con los datos establecidos. La precisión y fiabilidad del programa BEMCRACKER2D se demostró a través de estos ejemplos, mostrando que el programa puede predecir el comportamiento de la grieta con un alto grado de precisión. Los resultados destacan el potencial de BEMCRACKER2D como herramienta valiosa para el análisis de fracturas

cohesivas en materiales frágiles. Su capacidad para modelizar y predecir con precisión el crecimiento y desplazamiento de las grietas no sólo refuerza su aplicabilidad en la investigación académica, sino que también sugiere importantes aplicaciones prácticas en el diseño de ingeniería y las evaluaciones de seguridad.

Palabras clave: Fractura Cohesiva. Hormigón Plano. DBEM. BEMCRACKER2D

1 INTRODUCTION

The union between Linear Elastic Fracture Mechanics (LEFM) and Computational Mechanics brought notoriety to recurrent structural failures in brittle materials, even under the action of stresses lower than the design limit stresses, obtained normatively. Among this category of materials, concrete is the second most used material in the world, thus typified due to the arbitrary disposition of its granular constituents, presenting anisotropic composite behavior and also being subject to singular phenomena. Briefly, it is denoted as the agglutination of cementitious paste, aggregates and a transition zone between them formed under a high accumulation of water and converted into a region of already fragile material even before being mechanically requested (Gomes et al., 2016; Rodrigues, 2018; Rots and Blaauwendraad, 1989; Williams, 1957).

Its fracture mechanisms, representative for any brittle material, are respectively cleavage fracture and intergranular fracture. Briefly, brittle fracture by cleavage in crystalline materials is the most critical and, when considering the mechanisms of ductile fracture, the second most commonly seen in cases of fatigue or stress corrosion, revealing itself in situations where the grains are more fragile than the rest of the material and developing due to the breaking of the material's atomic bonds, which results in an agile faceted cracking process. Adversely, intergranular brittle fracture, often originating from external processes, manifests itself when the material surrounding the grain is more brittle than the material in the grain itself (Broek, 1984; Broek, 1988; Anderson, 2005; Carasek et al., 2018).

Still on fracture, concrete presents different behaviors depending on the cracking stage in which it is found. While not subjected to cracking, it can be accurately represented as an isotropic and homogeneous material with elastic-linear properties. In turn, after starting the process in an element subjected to different loadings, it continues to express elastic-linearity, but now with the presence of successive increments of tension and displacements on the crack, due to the propagation of microfaults, resulting in non-linear responses and consequent local failures in the part (Rots and Blaauwendraad, 1989; Rots et al., 1985).

In light of the constant advances in the analysis of cohesive fractures in recent years, this work will proceed using the adopted methodology in Leonel (2009) and Cordeiro (2015), which presents the two constitutive laws most commonly used for numerical analysis of cohesive fractures: linear and bilinear curves. This approach aims to provide a comprehensive understanding of the interference caused by cohesive fracture processes in concrete elements.

Therefore, this work seeks to present an analysis methodology utilizing the GUI BEMLAB2D and the processing software BEMCRACKER2D. By utilizing Linear Elastic Fracture Mechanics (LEFM) and the Dual Boundary Element Method (DBEM) DBEM (Delgado Neto, 2017; Gomes and Miranda, 2018). This combination has demonstrated excellent results for homogeneous materials in Boundary Element Method (BEM) applications without cracking, and DBEM in scenarios involving cracking. The capabilities of the BEMCRACKER2D software include the incremental determination of Stress Intensity Factors (SIFs), calculation of the J-integral, and prediction of the cracking direction using the Maximum Tangential Stress (MTS) criteria, along with stress and strain responses at each increment. Additionally, the GUI BEMLAB2D visualizes the deformed shape and crack path based on the data obtained (Delgado Neto, 2017).

This paper is organized as follows: Section 2 presents the main concepts and theories. Section 3 describes the methodology developed in this work. Section 4 showcases two applications of the defined methodology. Section 5 discusses and comments on the results obtained from the applications. Finally,

Section 6 provides concluding remarks on the effectiveness and performance of the employed methodology.

2 THEORETICAL BACKGROUND

2.1 OVERVIEW OF COHESIVE FRACTURE MECHANICS

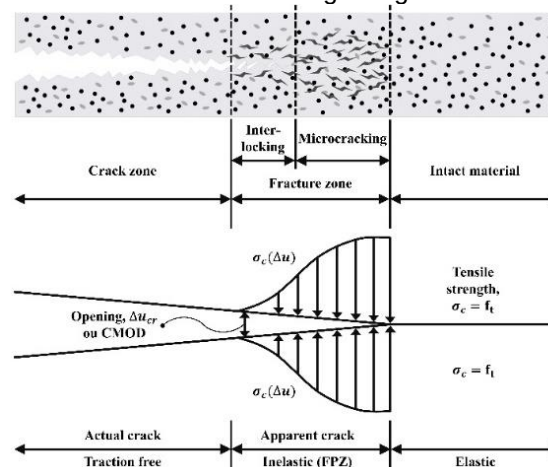
Adopting concrete as a reference material, since it is the most widely used brittle material in the world, one can observe the arbitrary composition of its granular constituents and its anisotropic composite behavior subject to singular phenomena, such as the size effect. Macroscopically referred to as a material derived from the union between cementitious paste (with or without additions or admixtures), aggregates and an interposed interface zone, also known as the transition zone, already weakened before being subjected to any mechanical action due to the presence of numerous micro-cracks especially originated by the high accumulation of water in the region (Rots and Blaauwendraad, 1989; Williams, 1957; Portela, Aliabadi e Rooke, 1992). These characteristics singularize the material behavior which, under tension application in a given element constituted by it, such as a test body, shows a limited number of cracks developed in arbitrary locations. However, if the resistance limit is exceeded, all additional deformation resulting from microcracks will be allocated within the process zone, denoted by the gradual decrease in stress values and a corresponding increase in deformation values, externalizing the occurrence of the coalescence phenomenon on microcracks and the consequent formation of a continuous macrocrack that will propagate until failure (Rots et al., 1985; Leonel, 2009; Cordeiro, 2015).

A well-established understanding refers to the emergence of a zone of inelastic processes (ZIP) ahead of the crack tip during the fracture process, initially possessing unknown proportions and shape, but controlled by the mechanical behavior of the material. Internal the zone of processes, there is a region of high stress concentration that depends on the ductility of the material under analysis, but which is capable of altering its initial shape, called the

fracturament process zone or simply of the fracturament zone, as shown in Figure 1(Rots et al., 1985; Leonel, 2009; Karihaloo, 1985; Oliveira, 2013). Brittle materials present a fracture zone that encompasses practically the entire ZIP, which has large dimensions when compared to those developed by the crack, and a significantly high cohesive strength in the particles ahead of the crack tip, allowing these materials to release energy initially through cracking and not through plasticization of the region (Oliveira, 2013; Hillerborg, Modéer, and Petersson, 1976).

In this way, Hillerborg, Modéer and Petersson (1976), state that the fictitious crack model shows good approximations in analyzing the influence of the fracture zone in the fracture process of brittle elements, making it possible to model cracking in brittle materials using the cohesive crack model, as shown in Figure 1.

Figure 1. Concrete crack modeling using cohesive elements.



Source: Durand and Silva, 2019 – Adapted.

Representing the strength of the cohesive zone by the cohesive stress, $\sigma_c(\Delta u)$, which attempts to close the crack and whose modulus is a function of the crack opening displacement, Δu or CMOD, being equal to the f_t of the material at the end of the ZPI and decreasing as it approaches the real crack tip (Leonel, 2009; Cordeiro, 2015; Hillerborg, Modéer, and Petersson, 1976; Saleh e Aliabadi, 1998). With this, for the analysis of cracks in brittle materials, it is obtained that by applying a load on a given element, the total energy rate released, G_q , is

dissipated both by a portion of energy to overcome the crack surface, G_{Ic} , and by a parcel of energy from the cohesive zone, G_{σ} , resulting in the cracking process (Leonel, 2009). Therefore, it follows that,

$$G_q = G_{Ic} + G_{\sigma} \quad (1)$$

From the equation (1), it becomes possible to perform two analyses, the one describing the crack propagation in brittle materials, and the other approximations of the energy dissipation mechanism to obtain different cracking prediction models. Assuming the model of a fictitious crack it is possible to consider that G_{Ic} is considerably small compared to G_{σ} , and one can admit that $G_{Ic} = 0$ when compared to the energy required to separate them.

2.2 LINEAR ELASTIC FRACTURE MECHANICS (LEFM)

This study took its first steps in the mid-1920s through the works of Griffith, followed by Williams and Irwin in the mid-1950s, and by Rice in the mid-1960s. The LEFM is a specialty of Fracture Mechanics responsible for the analysis of brittle materials, carried out in a highly simplified yet potentiated way, repairing the limitations concerning the presence of discontinuities of traditional concepts employed in strength of materials. It is applicable to any material, provided that its deformation is restricted to a considerably elastic region, i.e., a very small plastic region in relation to the total length of the crack or the dimensions of the part, allowing plastic phenomena to be disregarded (Anderson, 2005; Bazant and Planas, 1998; Kumar, 2009; Medina, 2014). This approach aims to determine its behavior, stress and displacement fields located near the crack tip, occurring through the use of characterization parameters such as SIF's and the energy release rate, $\mathcal{G}$. It should be noted that the adoption of LEFM assumes that stress, deformation and displacement fields can be determined by the conceptualization of SIF's near the crack tip (Bazant and Planas, 1998; Kumar, 2009; Medina, 2014; Portela, Aliabadi e Rooke, 1993; Serenza, Suarez and Rossi, 2020).

According to LEFM, the evolution of the cracking process is the predominant factor in the loss of integrity of fissured structural elements. In this context, the parameter K is presented as the sole characterizer of the stress field and, consequently, the behavior of a sharp crack on the element, which exhibits linear-elastic behavior until its critical value is reached, denoted as the Material Fracture Toughness, K_c , and unstable behavior after this limit, even without additional loads on the boundary, progressing until the fracture of the element occurs. Beyond this limit, the behavior becomes unstable, progressing until the fragile fracture of the element develops, even without any increase in loads on the boundary (Gomes et al., 2016; Carasek et al., 2018; Gomes and Miranda, 2018; Barsom and Rolfe, 1999; Dowling, 2013). In Anderson (2005) states that for occasions in which $K < K_c$, with values obtained numerically through (Broek, 1984), it is possible to determine the stress field at the tip of the crack of any structural element, directly correlating them with the specific K of each opening mode, represented here by m , as expressed below:

$$\lim_{r \rightarrow 0} \sigma_{ij}^{(m)} = \frac{K_m}{(2\pi r)^{1/2}} f_{ij}^{(m)}(\theta) \quad (2)$$

2.2.1 Crack Growth Criteria

The three main criteria for determining the direction of a given crack are the Maximum Circumferential Stress (MCS) criterion, the Maximum Potential Energy Release Rate (MPERR) criterion, and the Minimum Strain Energy Density (MSED) criterion. In compatibility with the adopted processing software, BEMCRACKER2D, this work will only address the first criterion, MCS, as this is its analysis methodology. Briefly, the method defines the crack direction as the one perpendicular to the maximum principal stress, $\sigma_{\theta max}$ and normal to the maximum circumferential stress, $\sigma_{r max}$, and to the planes where the shear stress, $\tau_{r\theta}$, is zero. The stresses at the crack tip are incrementally determined by summing the stresses individually linked to modes I and II, providing a trivial and

a non-trivial solution to be used for obtaining the values of θ , expressed in polar coordinates for the critical plane (Moura, 2019), as

$$\cos\left(\frac{\theta}{2}\right)\left[K_I \cos^2\left(\frac{\theta}{2}\right) - \frac{3}{2}K_{II} \sin\theta\right] = \sigma_\theta \sqrt{2\pi r} \quad (3)$$

$$\cos\left(\frac{\theta}{2}\right)\left[K_I \cos^2\left(\frac{\theta}{2}\right) + K_{II}(3\cos\theta - 1)\right] = 0$$

2.2.2 J-Integral

In Moura (2019), the author argues that the use of the J integral aims to form two paths, one closer to the crack and more complex, Γ_1 , and another, somewhat more distant and less complex, Γ_2 . Therefore, assuming a closed contour Γ in which, according to the principle of superposition, the sum of all its energy parcels will be zero, that on the crack face the traction vector, $T_i = d_y = 0$, results in the relations $J_1 = -J_2$ and $J_3 = J_4 = 0$, and that the stress and strain transformation equations are given by $E' = E$ and $E' = E/(1 - \nu^2)$, which results in the following relations (Rice, 1968):

$$J_1 = \frac{K_I^2 + K_{II}^2}{E'} \quad (4)$$

$$J_2 = -\frac{2K_I K_{II}}{E'}$$

2.3 DUAL BOUNDARY ELEMENT METHOD (DBEM)

In past decades, works with BEM addressed the application of only the boundary displacement equation to solve the primitive unknowns of displacement and traction of various problems. The present work brings a direct approach, avoiding the subdivision of the problems in sub-regions, using both integral boundary equations.

The dual boundary element method (DBEM) overcomes the crack-modelling difficulty by considering two independent equations, the displacement and the traction boundary integral equations, with the same integration path for each pair of coincident source points. Although the integration path is still the same, the boundary integral equations (BIE) are now distinct and therefore give rise to independent algebraic equations. This is the most general crack modelling procedure which allows a single-region boundary element analysis of cracked structures.

The BIE's will be briefly presented here, since they are implemented in BemCracker2D program (Rice, 1968) and are beyond the scope of this work. For the displacements BIE, considering the inelastic terms, we have,

$$c_{ij}(\xi)\dot{u}_j(\xi) = \int_{\Gamma} u_{ij}^*(\xi, x)\dot{p}_j(x)d\Gamma(x) - \int_{\Gamma} p_{ij}^*(\xi, x)\dot{u}_j(x)d\Gamma(x) + \int_{\Omega} u_{ij}^*(\xi, x)\dot{b}_j(x)d\Omega(x) \quad (5)$$

Where:

Γ and Ω are, respectively, the contour and domain of the whole body; u_{ij}^* and p_{ij}^* are the well-known fundamental Kelvin solutions and represent the weighting fields of displacement and traction in an elastic and homogeneous body subjected to a unitary force; and the traction $\dot{p}_j$ and the body forces $\dot{b}_j$ are fictitious (they depend on inelastic deformations), but the displacements $\dot{u}_j$ are real.

For the traction BIE, obtained by differentiating Equation **Erro! Fonte de referência não encontrada.**, we have,

$$\begin{aligned} \frac{1}{2} \dot{p}_j(\xi) = & n_i(\xi) \int_{\Gamma} U_{ijk}^*(\xi, x) \dot{p}_k(x) d\Gamma(x) \\ & - n_i(\xi) \int_{\Gamma} P_{ijk}^*(\xi, x) \dot{u}_k(x) d\Gamma(x) \\ & + n_i(\xi) \left[\int_{\Omega} U_{ijk}^*(\xi, x) \hat{b}_k(x) d\Omega(x) + \frac{1}{2} C_{ijkl} \dot{\epsilon}_{kl}^a \right] \end{aligned} \quad (6)$$

Where:

n_i represents the i-th unit component of the external normal to the boundary, at ξ point;
 C_{ijkl} represents the fourth order isotropic tensor of the elastic constants.

The integrals of Cauchy's principal value and Hadamard are equivalent to the first and second order integrals of finite parts. The first-order integral refers to the displacement equation that satisfies the solution for any continuous or discontinuous boundary element, but the second-order integral only presents satisfactory results for discretized boundaries through discontinuous boundary elements. Thus, Equations **Erro! Fonte de referência não encontrada.** and **Erro! Fonte de referência não encontrada.** presented in the works of Gomes et al. (2016) and Rodrigues (2018) together with the discretization strategy for internal and edge cracks presented by Portela, Aliabadi and Rooke (1993) and implemented in BemCracker2D (Gomes and Miranda, 2018), will be applied in this work.

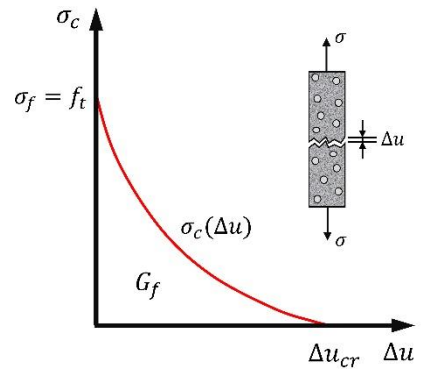
2.4 CONSTITUTIVE LAWS: LINEAR AND BILINEAR CURVES

These models arise in response to the need to adapt methodologies related to the determination of the ZPI, presenting approximations over the behavior of the fracture zone for the most diverse materials. Summarily, the fracture zone subjected to cohesive stress, σ_c , which tends to join the faces of the crack acting in a small portion of the crack tip, denoted as the cohesive zone, where there are still mechanisms of the fracturing zone capable of transmitting stresses. Through this, this allows for the determination of the fracture energy to

be employed so that occurs for the process of softening of concrete under tensile stresses. This process requires the performance of a uniaxial tensile test of concrete for plotting the stress-displacement graph, representative of the softening zone of concrete, as seen in Figure 2.

$$G_f = \int_0^{\Delta u_{cr}} \sigma_c(\Delta u) d\Delta u \quad (7)$$

Figure 2. Curve of cohesive stress σ_c versus elongation of the fractured section, or Δu , for concrete.



Source: Cordeiro, 2015 – Adapted.

2.4.1 Fictitious Crack Model

This model has as its main criterion the delimitation of the beginning of the fracturing zone, allocating it at the moment when the maximum principal stress at the tip of the cohesive crack and the theoretical tensile strength of the material become equal, provided that the crack opening does not exceed its limit value and, consequently, becomes free of cohesive forces (see Figure 1).

During this process, cracking occurs perpendicular to the direction of action of the maximum principal stress and, even if partially damaged, the fracturing zone still has the capacity to transfer stresses as a function of the distance between the crack faces, preserving the linear-elastic properties in the material outside of it. There are several methods used to obtain the cohesive forces acting over the fracturing zone, being practiced in this work adopts the two constitutive laws most commonly used in the literature: linear and bi-linear, whose purpose is to correlate, with a certain degree of accuracy, the values of f_t and Δu .

Some important relationships are noticed, such as the equality between the maximum load transmitted by an intact material, σ_f , and f_t , which is a material

parameter obtained through the test. It is also observed that the softening curve is delineated by the path of σ_c as a function of Δu , $\sigma_c(\Delta u)$, which, upon reaching its critical value, Δu_{cr} , marks the moment when there is no more transmission of forces between the faces of the fissure, i.e., $\sigma_c(\Delta u_{cr}) = 0$. Finally, another material parameter is expressed, also obtained through the tensile test, named as energy required for fracture in mode I, G_f , determined as the value corresponding to the area under the softening curve.

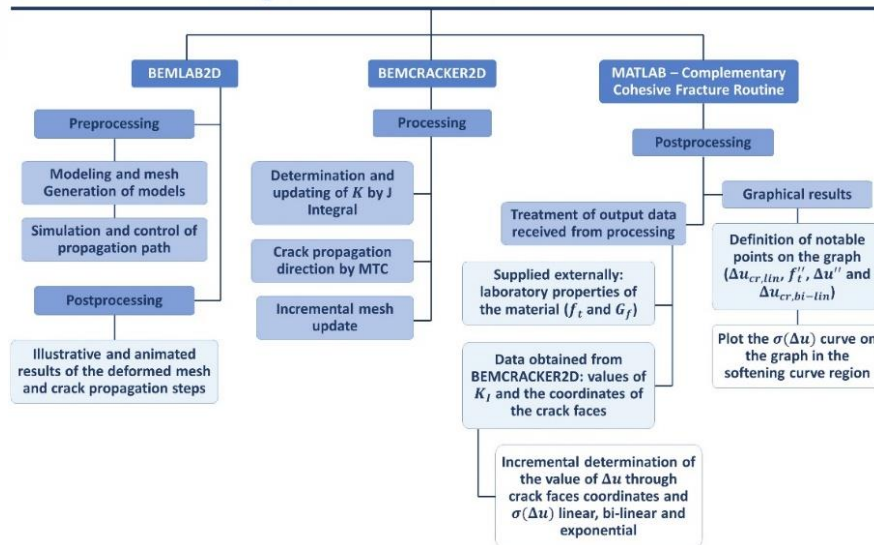
3 MATERIAL AND METHOD

The main objective of this work is to present a methodology proposal for simulation and evaluation of cohesive fracture, using an interaction process between the BEMLAB2D and BEMCRACKER2D programs, consisting of two stages, as illustrated in the diagram of figure 3.

Step 1: Data Modeling and Mining

This stage is characterized by the use of the software BEMLAB2D for the pre-processing, and BEMCRACKER2D solver. Therefore, the pre-processing phase begins by defining the model and performing its modeling in BEMLAB2D, through tools available in the user interface, followed by defining the zones, mesh and boundary conditions.

Figure 3. Applied Methodology.
Methodology for Propagation Analysis of Multiple Critical Cracks and Cohesive Fracture
Using BEMLAB2D and BEMCRACKER2D Software



Source: The authors (2024).

Step 2: Obtaining Notable Points

The post-processing stage is started with files resulting from the previous stage. This stage is fractioned into two phases, the first one consists of determining the results related to the deformed mesh and the crack propagation path, which requires returning to BEMLAB2D. On the other hand, the second phase involves the cohesive analysis of the element, requiring the acquisition of data as the incremental displacements of the crack faces, as well as their respective K parameters, also incremental, of displacements, DX , given that $\Delta u = DX_{dir} - DX_{esq}$, repeating the same data acquisition process for each increment.

From of this information, it becomes possible to apply the BEMLAB2D again for analyze the two constitutive laws belonging to the cohesive fracture model, which also requires the laboratory values of f_t and G_f , characteristic for each problem, employ in the determine the incremental curve of the cohesive stress as a function of crack opening displacement, $\sigma_c(\Delta u)$, denoted as the material softening curve. It becomes evident that due to the application of the linear and bi-linear constitutive laws, the resulting cohesive analysis graph will present a curve for each of the represented laws, which represent the same element, however with different levels of precision and possibilities of applications.

3.1 DESCRIPTION OF BEMCRACKER2D AND BEMLAB2D GUI

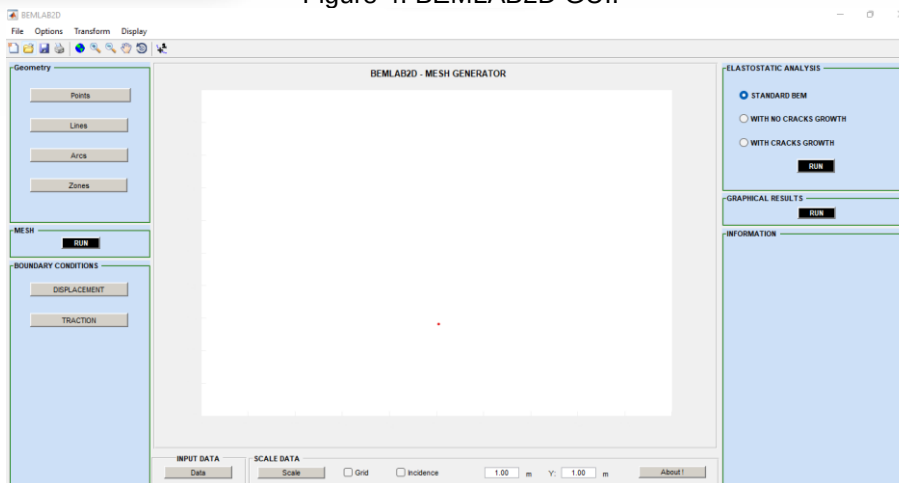
BEMCracker is a program developed in C++ by (Gomes et al., 2016) that allows the analysis of two-dimensional elastostatic problems using the Boundary Element Method. The program has three processing modules, which are:

- Standard BEM – Module I;
- Non-propagating DBEM – Module II;
- Propagating DBEM – Module III.

In Module III, which deals with the analysis of propagating cracks, a stress analysis is performed using BEM, an evaluation with Stress Intensity Factors (SIFs) using the J-Integral, an assessment of crack growth direction following the three criteria mentioned in the previous section, fatigue life evaluation (performed according to Paris' Law), coalescence analysis in multiple crack scenarios, among others. The implementation proposed in this work will be carried out in this module, where several methods has been developed to simulate cohesive fracture in concrete.

BEMCracker2D uses a GUI called BEMLAB2D, a graphical interface developed in MATLAB, which serves for pre-processing and post-processing. It was developed by Delgado Neto (2017), aiming to generate input data through problem modeling with the help of dialog boxes, as well as to provide visualization of two-dimensional elements. Figure 4 shows the BEMLAB2D interface.

Figure 4. BEMLAB2D GUI.



Source: The authors (2024).

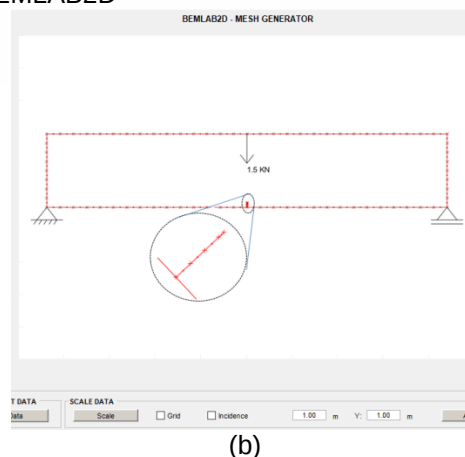
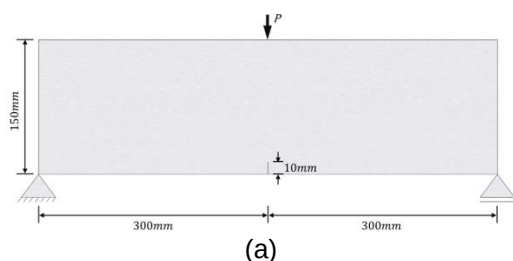
4 APPLICATION EXAMPLES

Using the BEMLAB2D GUI tools for modeling, mesh generation and visualization of results, and the BemCracker2D program for incremental load analysis, we explored several scenarios to validate our approach. The following examples show the fracture behavior of the material through load x deflection curves (LINEAR and BILINEAR), as well as propagation and deformed shape, for simple concrete beams at three flexion points, providing a detailed and practical view of the application of our methodology.

- Example 1

It is a concrete beam at three points with a lower centered crack and a point load also centered on the upper boundary, as illustrated in Figure 5, where: a) physical model; and b) discrete model of boundary elements.

Figura 5. Plain concrete beam: a) physical model (Adapted from Mojiri, 2010); b) boundary mesh model by BEMLAB2D



Source: The authors (2024).

The input parameters are described in Table 1.

Table 1. Input parameters for example 1.

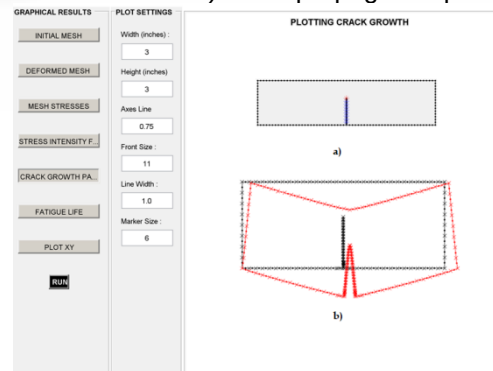
Variable	Value	Unit
Problem	Plane Stress State (PSS)	
E, v, Load	36500, 0.1, 1.5	MPa, -, kN
GF, FT	Varied range, 3.19	Nm ⁻¹ , MPa

Source: The authors (2024).

Figure 6 shows the boundary element mesh with the crack propagation path and its deformed shape for 8 increments.

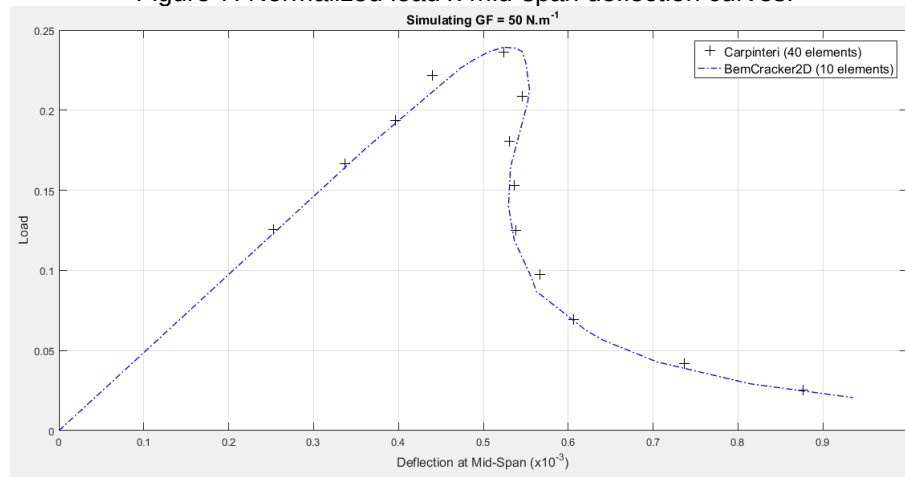
The beam is analyzed for the fracture energy value $GF = 50 \text{ Nm}^{-1}$, considering the plane stress state (PSS) and a point load. In Figure 7 we have the normalized load vs. mid-span deflection curves, in which we observe a good convergence to that due to (Carpinteri and Colombo, 1989). In this, we can highlight that they used 40 elements to simulate the propagation path in the mid-span, compared to 10 linear elements of BemCacker2D.

Figure 6. Boundary element mesh: a) crack propagation path; b) deformed shape.



Source: The authors (2024).

Figure 7. Normalized load x mid-span deflection curves.

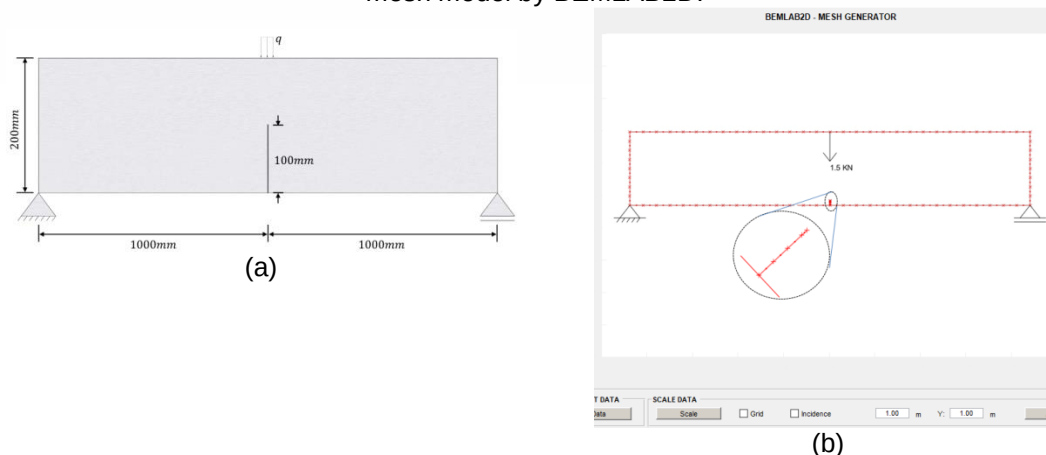


Source: The authors (2024).

- **Example 2**

It is a cracked concrete beam subjected to bending at three points, tested experimentally and numerically by Petersson (1981). The pre-crack size (100 mm) is half the height of the beam as illustrated in the physical model in Figure 8(a) and the boundary element mesh in Figure 8(b).

Figura 8. Plain concrete beam: a) physical model (Adapted from Petersson, 1981); b) boundary mesh model by BEMLAB2D.



Source: The authors (2024).

The input parameters are described in Table 2.

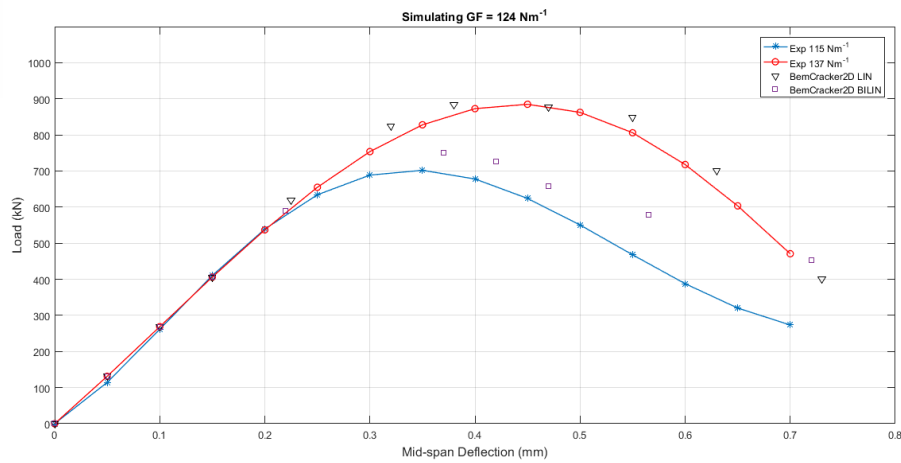
Table 2. Input parameters for example 2.

Tipo	Valor	Unidade
Problem	Plane Stress State (PSS)	
E, v, Load	30000, 0.2, 1	MPa, -, MPa
GF, FT	Varied range, 3.33	Nm ⁻¹ , MPa

Source: The authors (2024).

The beam is analyzed for the minimum and maximum values of GF (115 and 137 Nm⁻¹) obtained experimentally and according to Petersson (1981), considering the plane stress state. In Figure 9, we have the load vs. deflection curves, both LINEAR and BILINEAR, for GF equal to 124 Nm⁻¹, from which we observe that the BILINEAR presents better results compared to the average (126 Nm⁻¹), being within the experimental envelope.

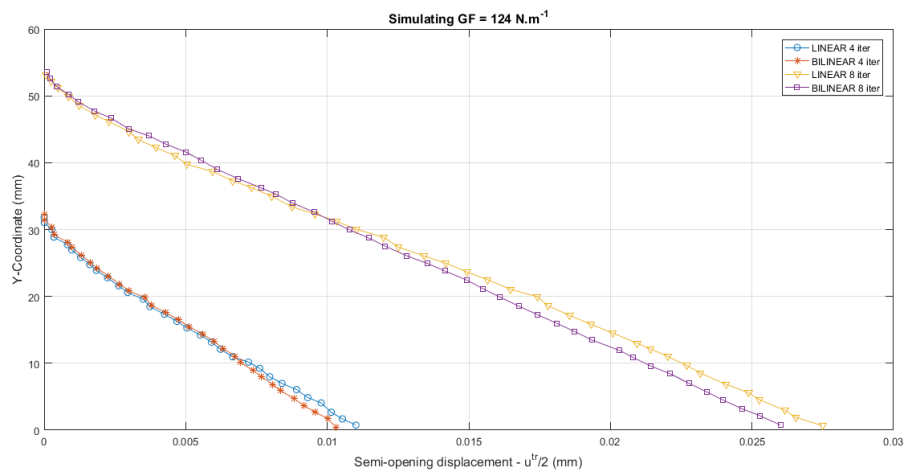
Figure 9. Load x mid-span deflection curves.



Source: The authors (2024).

In Figure 10, we present the crack displacement opening along the beam's axis of symmetry, therefore, $u^{\text{tr}}/2$ for each increment (Y coordinate) with 4 and 8 iterations, respectively, considering $GF = 124 \text{ Nm}^{-1}$.

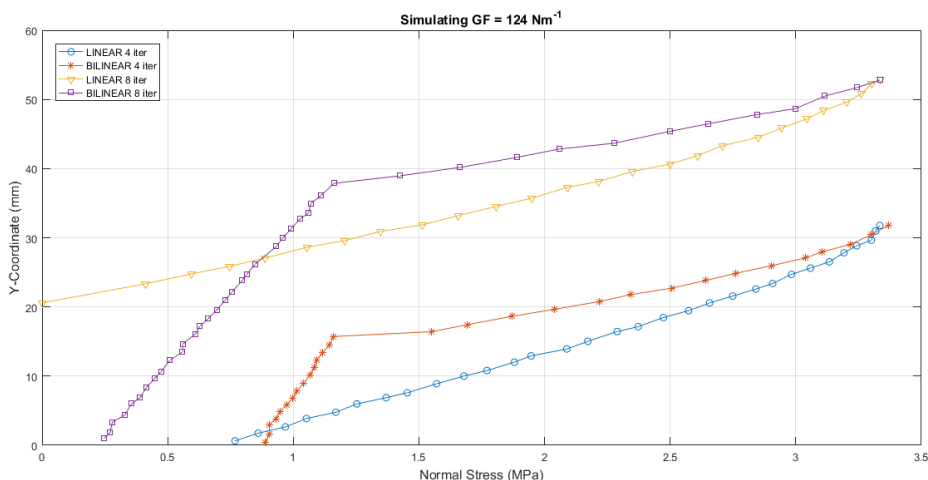
Figure 10. Crack semi-open displacement along line of symmetry for 4 and 8 iterations.



Source: The authors (2024).

In Figure 11, we have the normal stress along the beam's axis of symmetry plotted against the Y coordinate for each step at 4 and 8 iterations, respectively, considering $GF = 124 \text{ Nm}^{-1}$, which was the closest value to the experimental curve. It is observed in Figure 11 that the crack is fully loaded by iteration 4 for both models, but by iteration 8, the traction-free crack only appears in the LINEAR model.

Figure 11. Stresses along line of symmetry for 4 and 8 iterations.



Source: The authors (2024).

5 RESULTS AND DISCUSSION

The two models, described in the previous section, were implemented according to the methodology developed in this study and previously outlined. Initially, the analysis of results was conducted through visual inspection of output data provided by the BEMCRACKER2D processing software. This included incremental stress and strain responses, mesh geometry updates, as well as incremental values of crack face coordinates and the stress intensity parameter K at the crack tip. The structural element and crack behavior could thus be assessed directly from the output data, eliminating the constant need for graphical representations.

Upon confirming the numerical results' approximation to reference values, the BEMLAB2D graphical user interface (GUI) was utilized to obtain graphical representations of the crack propagation path and the deformed shape of the beam.

The analyses of cohesive fracture using two constitutive laws, implemented through MATLAB routines independent of BEMLAB2D and BEMCRACKER2D, are evidenced by the constant softening curves displayed in the graphs. These curves visually highlight characteristic and expected moments, including: the initial and final points of the linear curve, the initial point, mid-vertex,

and final point of the bilinear curve, with the latter particularly close to experimental curves.

Furthermore, there is noticeable variation in data point concentration along the curves, characterized by the cessation of initially inserted points and the incremental analytical addition of points based on the $\sigma_c(\Delta u)$ curve trend. This approach allows for a denser representation of behavior up to the stability point, reached when $\Delta u = \Delta u_{cr}$, and consequently, $\sigma_c(\Delta u_{cr}) = 0$, for any $\Delta u > \Delta u_{cr}$.

6 CONCLUSIONS

This study investigated two computational models for crack propagation analysis in beams, using the BEMCRACKER2D software and the BEMLAB2D graphical interface. The adopted methodology allowed for a detailed analysis of the results, including incremental stresses, strains, as well as crack face coordinates and stress intensity parameters at the crack tip. The research question, which sought to determine the effectiveness of the proposed models in predicting crack behavior in beams, was answered positively, with the obtained numerical results demonstrating a good approximation to reference values and validating the effectiveness of the models.

Furthermore, the application of Cohesive Fracture laws, implemented through independent MATLAB routines, revealed consistent softening curves in the analyzed graphs. These curves highlighted characteristic moments such as the initial and final points of linear curves, as well as the initial, mid-vertex, and final points of bilinear curves, with the latter showing closer approximation to experimental curves.

Variation in data point concentration along the curves was also observed, reflecting the necessary density to capture behavior up to the stability point, where crack closure occurs. This stability point is identified when the displacement opening reaches its critical value, resulting in zero cohesive stress.

The results of this study provide valuable insights into the structural behavior of beams subjected to crack propagation, validating the numerical approaches used. For society, these results can contribute to the development of

safer and more durable structures, preventing catastrophic failures and reducing maintenance costs. In academia, this study advances the understanding of fracture processes in structural materials, providing a solid foundation for future research and potentially influencing structural design and analysis guidelines.

However, this research has some limitations, such as the reliance on numerical simulations that may not capture all the complex aspects of real crack behavior in the field. Recommendations for future work include validating the models with a wider range of materials and loading conditions, as well as developing complementary experimental techniques to enrich the data and enhance the accuracy of the simulations.

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