



TWO-PHASE FLOW PATTERN ANALYSIS FROM PRESSURE AND VIBRATION SIGNATURE

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**Doctoral Thesis
Mechanical Sciences**

UNIVERSITY OF BRASILIA

**Technology Faculty
Mechanical Engineering Department**

UNIVERSITY OF BRASILIA
TECHNOLOGY FACULTY
MECHANICAL ENGINEERING DEPARTMENT

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signature

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Daniely Amorim das Neves

Dedicated to God, my parents, and my brother.

"O Altíssimo conhece toda a ciência e observa os sinais dos tempos, anunciando as coisas passadas e futuras e revelando os vestígios das coisas escondidas. Ele dispôs em ordem as maravilhas de sua sabedoria, porque só Ele existe desde sempre e para sempre. Como são agradáveis todas as suas obras, ainda que delas se veja apenas uma faísca". Ecclesiasticus 42:16-22.

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Abstract

TWO-PHASE FLOW PATTERN ANALYSIS FROM PRESSURE AND VIBRATION SIGNATURE

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Supervisor: Adriano Todorovic Fabro, Dr. (ENM/ UnB)

Doctoral Thesis

The application of indirect approaches for characterising flow patterns and their parameters offers advantages by eliminating the subjectivity inherent in traditional inspection-based methodologies. In addition to being potentially cost-effective and easy to install, these approaches also enable integration with other supervisory systems and the Internet of Things. In this context, this work proposes methodologies for identifying flow patterns and characterising intermittent two-phase flow in horizontal pipes through non-invasive/non-intrusive pressure and acceleration sensors. For this purpose, signal processing techniques are employed in conjunction with physical models. Acceleration, dynamic pressure, and void fraction measurements are obtained from a horizontal line of liquid-gas two-phase flow. 23 experimental points are acquired in the stratified-smooth, stratified-wavy, slug, and dispersed bubble patterns. Fluid-structure coupling is investigated between pressure and structural waves, concentrating on the cut-on frequencies of the circumferential pipe wave modes. A methodology is proposed to demodulate acceleration signals around these cut-on frequencies, which can be estimated analytically. From the demodulated signal, it was possible to classify the flow pattern using values of Hurst exponent and Pearson and Spearman coefficients. Additionally, velocity and frequency parameters in Slug patterns were estimated with errors below 20%. Furthermore, this work proposes an approach to identify the pattern of two-phase flow through non-invasive pressure measurements. Flow patterns are classified based on dimensionless thresholds from a nonlinear analysis of time series, namely the correlation dimension, Lyapunov, and Hurst exponents. The dynamics of the 23 experimental points are investigated, and the results indicate that the system response is chaotic for all flow patterns due to the positive sign of the Lyapunov exponent. From the Hurst exponent, it was possible to separate oscillatory flow patterns, such as slug patterns, from non-oscillatory flow patterns, such as stratified. The correlation dimension separated smooth and wavy stratified patterns from intermittent and dispersed bubble

patterns. Subsequently, it is shown that these parameters form a multidimensional map that clusters the experimental points according to the dynamics of each flow pattern. In light of this, these results open new and interesting possibilities for applications in complex industrial systems.

Key-words: two-phase flow, flow pattern identification, fluid-structure interaction.

Resumo

ANÁLISE DE ESCOAMENTO BIFÁSICO A PARTIR DA ASSINATURA DE PRESSÃO E VIBRAÇÃO

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Orientador: Adriano Todorovic Fabro, Dr. (ENM/ UnB)

Programa de Pós Graduação em Ciências Mecânicas

A aplicação de abordagens indiretas para a caracterização de padrões de escoamento e seus parâmetros apresenta vantagens ao eliminar a subjetividade inerente às metodologias tradicionais baseadas em inspeção. Além de serem potencialmente de baixo custo e de fácil instalação, essas abordagens também permitem a integração com outros sistemas supervisórios e a Internet das Coisas. Neste contexto, este trabalho propõe metodologias para a identificação de padrões de escoamento e a caracterização do escoamento bifásico intermitente em dutos horizontais por meio de sensores não-invasivos/não-intrusivos de pressão e aceleração. Para isso, são empregadas técnicas de processamento de sinais em conjunto com modelos físicos. Medições de aceleração, pressão dinâmica e fração de vazios são obtidas a partir de uma linha horizontal de fluxo bifásico líquido-gás. E são adquiridos 23 pontos experimentais nos padrões estratificado-liso, estratificado-ondulado, slug e bolhas dispersas. O acoplamento fluido-estrutura é investigado entre ondas de pressão e estruturais, concentrando-se nas frequências de corte dos modos de onda circunferencial do duto. Propõe-se, assim, uma metodologia para demodular os sinais de aceleração em torno dessas frequências de corte, que podem ser estimadas analiticamente. A partir do sinal demodulado, foi possível classificar o padrão de fluxo usando valores de expoente de Hurst e coeficientes de Pearson e Spearman. Além disso, foi possível estimar parâmetros de velocidade e frequência nos padrões Slug com erros inferiores a 20%. Além disso, este trabalho propõe uma abordagem para identificar o padrão de escoamento bifásico por meio das medições de pressão não invasivas. Os padrões de fluxo são classificados com base em limiares adimensionais provenientes de uma análise não linear de séries temporais, nomeadamente a dimensão de correlação, expoentes de Lyapunov e Hurst. Os resultados indicam que a resposta do sistema é caótica para todos os padrões de fluxo devido ao sinal positivo do expoente de Lyapunov. A partir do expoente de Hurst, foi possível separar os padrões de fluxo oscilatórios, como os padrões de slug, dos padrões de fluxo

não oscilatórios, como os estratificados. A dimensão de correlação separou os padrões estratificados dos padrões de bolhas intermitentes. Posteriormente, mostra-se que esses parâmetros formam um mapa multidimensional que agrupa os pontos experimentais de acordo com a dinâmica de cada padrão de fluxo. Diante disto, esses resultados abrem novas e interessantes possibilidades para aplicações em sistemas industriais complexos.

Palavras-chaves: escoamento bifásico; padrão de escoamento; propagação de onda; interação fluido-estrutura.

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List of abbreviations

AN	Annular
ACF	Autocorrelation Function
AGA	American Gas Association
AMI	Average Mutual Information
API	American Petroleum Institute
BCC	Basic Cross-Correlation
BSD	Bubble Size Distribution
CEPETRO	Centro de Estudos de Petróleo
CD	Correlation Dimension
CNN	Convolutional Neural Networks
CPSD	Cross Power Spectral Density
DAQ	Data Acquisition Module
DB	Dispersed Bubbles
DP	Differential Pressure
DFT	Discrete Fourier Transform
EB	Elongated Bubble
FDCN	Fluid Dynamic Complex Network
FIV	Flow Induced Vibration
FNN	False Nearest Neighbor
FRF	Frequency Response Function
FSI	Fluid Structure Interaction

GCC	Generalized Cross-Correlation
HHT	Hilbert-Huang Transform
KDE	Kernel Density Estimation
KNN	K-Nearest Neighbors
LABPETRO	Laboratório Experimental de Petróleo
LDV	Laser Vibrometers
LP	Liquid Piston
ML	Maximum Likelihood
NI	National Instruments
PDF	Probability Density Function
PSD	Power Spectral Density
RMS	Root Mean Square
SAFE	Semi-Analytical Finite Elements
SAM	Segment Anything Model
SVM	Support Vector Machine
SNR	Signal Noise Ratio
SS	Stratified-Smooth
SISO	Single-Input-Single-Output
STFT	Short-Time Fourier Transform
SW	Stratified-Wavy
TB	Taylor Bubble
UNB	Universidade de Brasília
UNICAMP	Universidade de Campinas
VVG	Vector Visibility Graph
WA	Wavy-Annular

List of symbols

Latin letters

index $_G$	Gas phase [-]
index $_L$	Liquid phase [-]
a	Diameter of pipe [m]
A	Cross-sectional area [m]
A_G	Cross-sectional area of gas [m]
A_L	Cross-sectional area of liquid [m]
Ac	Acceleration signal [m/s]
c	Correlation dimension [m]
c_b	Propagation velocity [m/s]
c_G	Velocity of the sound of the gas [m/s]
c_L	Velocity of the sound of the liquid [m/s]
c_p	kinematic velocity [m/s]
d	Diameter [m]
d_a	Original dimension [-]
d_{avg}	Average diameter [m]
d_b	Bubble diameter [m]
d_{BK}	Broadkey diameter bubble [m]
d_e	Embedding dimension [-]
d_G	Hydraulic diameter of gas phase [m]
d_H	Hinze bubble diameter [m]

d_L	Hydraulic diameter of liquid phase [m]
dp	Pressure drop [Pa]
ds	Infinitesimal element [-]
E	Modulus elasticity [Gpa]
EI	Flexural stiffness [Pa]
f_c	Critical frequency [Hz]
f_{coin}	Coincidence frequency [Hz]
f_{BK}	Broadkey frequency [Hz]
f_H	Hinze frequency [Hz]
f_F	Fanning friction factor [-]
f_{nb}	Frequency of oscillation of the bubbles [Hz]
f_n	Cut-on frequency [Hz]
f_{RP}	Rayleigh-Plesset frequency [Hz]
f_{ring}	Ring frequency [Hz]
f_s	Slug frequency [Hz]
Fr	Froude number [-]
g	Gravity acceleration [m/s^2]
H	Frequency Response Function [Hz]
H_{re}	Receptance [Hz]
H_{mo}	Mobility [Hz]
H_{ac}	Accelerance [Hz]
H_{LLS}	Liquid holdup in the stratified region [-]
H_{LTB}	Liquid holdup in the stratified region [-]
H_L	Holdup [-]
h	Hurst exponent [-]
I	Mutual Information [bits/s]
J_m	Superficial velocity of the mixture [m/s]

J_{sg}	Superficial velocity of the gas phase [m/s]
J_{sl}	Superficial velocity of the Liquid phase [m/s]
k	Stiffness [N/m]
$k_{1,2}$	Linear springs [N/m]
k_{θ_1, θ_2}	Torsion springs [N/m]
k_m	Wavenumber [rad/m]
k_{cs}	Wavenumber in helical direction [rad/m]
k_z	Wavenumber in axial direction [rad/m]
k_s	Wavenumber in circumferential direction [rad/m]
L_{LP}	Liquid piston length [m]
L_S	Liquid piston length [m]
L_U	Unit cells length [m]
L_{TB}	Bubble length [m]
L_F	Taylor bubble length [m]
M	Bending moment [N/m]
m	Axial mode [-]
N_b	Number of bubbles [-]
N_{seg}	Number of blocks [-]
p	Pressure [Pa]
P_{re}	Pressure signal [Pa]
q	Total volumetric flow rate [m ³ /s]
q_L	Volumetric flow of liquid phase [m ³ /s]
q_G	Volumetric flow of gas phase [m ³ /s]
R	Radius [m]
$R_{1,2,3,4}$	Resistive sensors [-]
Re	Reynolds number [-]
R_{xx}	Autocorrelation function [-]

R_{xy}	Cross-correlations function [-]
S_{xx}	Power spectral density function [-]
S_{xy}	Cross-spectral density function [-]
S_f	Slip factor [-]
S_I	Interfacial area [-]
S_G	Interfacial area of gas phase [-]
S_L	Interfacial area of liquid phase [-]
t	Time [s]
T	Temperature [°C]
T_∞	Temperature constant [°C]
U_F	Flow velocity [m/s]
V	Shear force [N]
V_G	Actual velocity of the gas phase [m/s]
V_L	Actual velocity of the liquid phase [m/s]
V_{GLS}	Velocity of the gas phase in the liquid piston [m/s]
V_{GTB}	Taylor bubble velocity in the stratified region [m/s]
V_{LLS}	Velocity of the liquid phase in the liquid piston [m/s]
W	Total mass flow rate [kg/s]
W_L	Mass flow rate of liquid phase [kg/s]
W_G	Mass flow rate of gas phase [kg/s]

Greek letters

α	Void fraction [-]
α_{LP}	Void fraction of liquid piston [-]
α_{TB}	Void fraction of Taylor bubble [-]
γ	Coherence function [-]
ζ	Spearman coefficient [-]
λ	Lyapunov exponent [-]

λ_G	No-slip void fraction [-]
μ	Mean [-]
μ_L	Viscosity of the liquid phase [cp]
μ_G	Viscosity of the gas phase [cp]
μ_M	Viscosity of the mixture phase [cp]
ρ	Pearson coefficient [-]
ρ_F	Density of fluid [kg/m^3]
ρ_G	Density of the gas phase [kg/m^3]
ρ_L	Density of the liquid phase [kg/m^3]
ρ_p	Specific mass of pipe [kg/m^3]
σ	Surface tension [N/m]
τ	Time delay [s]
τ_w	Shear stress [N/m^2]
θ	Angle [deg]
ξ_1	Skewness [-]
ξ_2	Kurtosis [-]
ω	Radial displacement [-]
Ω_n	Non-dimensional frequency [-]

1 Introduction

1.1 Introduction

The phenomenon of internal two-phase flow in pipeline systems is observed in various industrial applications and plays a crucial role in the production and transportation of oil and gas in the petrochemical industry (XU et al., 2023; GONG et al., 2023; TERENCE et al., 2019). This phenomenon is characterized by the presence of different spatial and geometrical configurations between the phases, known as flow patterns (ISHII; HIBIKI, 2011). In industrial applications, the quest for specific flow pattern configurations is a common strategy to enhance mixing and phase separation processes, aiming to maximize efficiency and ensure desired outcomes. Examples include the need for homogeneity in mixing during chemical procedures, the simplification of effective phase separation, the optimization of heat transfer in thermal systems, the regulation of chemical reactions to enhance process efficiency, the minimization of pressure losses to improve energy efficiency, and the mitigation of vibrations and wear in industrial machinery (GONG et al., 2023; PEDERSEN; DURDEVIC; YANG, 2015; VIEIRA et al., 2021). Therefore, the identification and characterization of the flow pattern play a crucial role in optimizing industrial processes, directly impacting the quality and efficiency of operations.

Three general topologies of two-phase flow can be usefully identified: (i) separate phases flow, where the different phases flow parallel separated by an interface, (ii) dispersed flow, where drops or bubbles (the disperse phase) are distributed in a continuous phase and (iii) intermittent flow, which presents characteristics of both previous types (ISHII; HIBIKI, 2011; SHOHAM, 2006). Usually, flow pattern identification and characterization are carried out through direct methods such as using high-velocity cameras (HANAFIZADEH; GHANBARZADEH; SAIDI, 2011), x-ray devices (HEINDEL, 2011), and electrical capacitance tomography (WANG; ZHANG, 2009). However, the inherent subjectivity in direct interpretation can lead to divergent observations, resulting in imprecise identification of flow patterns. Moreover, the use of direct methods can be limited by the need for direct access to the region of interest, which may not be practical in confined industrial environments. These limitations underscore the importance of developing more robust and automated indirect methods for the accurate identification and characterization of flow patterns in two-phase flows.

Faced with the challenges posed by direct measurements in identifying and characterising flow patterns, several authors explore techniques that rely on indirect measurements. [Hubbard \(1966\)](#), [Matsui \(1986a\)](#), [Ding et al. \(2007a\)](#) and [Santoso, Indarto e Thomas \(2012\)](#) used non-intrusive pressure sensors to identify flow patterns. These authors employ different statistics and signal processing techniques, like Power Spectral Densities (PSD), Probability Density Function (PDF) and statistical parameters, such as mean, standard deviation, kurtosis, and skewness. Vibration signals have been extensively studied in self-excited piping systems with two-phase flow ([RIVERIN; PETTIGREW, 2007](#); [MIWA; MORI; HIBIKI, 2015](#); [ORTIZ-VIDAL; MUREITHI; RODRIGUEZ, 2017](#)), an area known as Flow Induced Vibration (FIV). This method offers the significant advantage of being non-invasive and non-intrusive, providing an effective approach to monitoring the dynamic behavior of these systems. [Carvalho, Figueiredo e Serpa \(2019\)](#) utilized vibration signals to identify slug and churn flow patterns using parameters such as Root-Mean-Square (RMS) and the Pearson coefficient. Other researchers use Machine Learning algorithms to identify flow patterns, employing multiscale wavelet packet transform, Hilbert-Huang Transform (HHT) Short Time Fourier Transform (STFT) as features ([HUA et al., 2010](#); [CHEN et al., 2023](#)). However, these works do not seek to investigate the complex and rich dynamics of each flow pattern, which involves multiple space-time scales, nor do they provide dimensionless estimators for the flow pattern characterisation. Some techniques are used to understand the dynamics of two-phase flow in terms of fractal and chaos theories. Fractal analysis, which assumes that system dynamics are inherently random, was applied to pressure drop signals by [Franca et al. \(1991a\)](#). The authors viewed the results as promising but also noted that more studies were necessary. Chaos theory studies the irregular behaviour of complex systems generated by nonlinear deterministic interactions, that is, future behaviour described by initial conditions. Deterministic chaos analysis was performed in pressure fluctuations by [Drahoš et al. \(1996\)](#) and the authors could elucidate some aspects of plug-to-slug transition and slug-to-annular transitions, but the study did not provide a methodology to objectively classify the different flow patterns.

In this context, vibration-based measurement of two-phase flow in pipes has the potential for standing out as a non-invasive/non-intrusive approach and, consequently, multiphase-flow induced vibration in pipes has receiving increasing attention in recent years, e.g. ([LANNES; GAMA, 2018](#); [MOHMMED et al., 2020](#)). It is remarkable that, although the dynamics of single-phase flow induced vibration have been well consolidated in the last decades ([PAIDOUSSIS, 2008](#)), it still is considered a fertile dynamic problem ([PAIDOUSSIS, 2022](#)). Very few research has been done at the structural wave behaviour of higher frequency dynamics of pipes conveying two-phase flow ([MATOS et al., 2022b](#); [MATOS et al., 2022d](#)). Unlike the typical modal approach, wave-based dynamics relies on the local properties of the pipe, which is very appealing considering the varying operational condition at industrial applications. An important behaviour is observed when an acoustic frequency is equal or close to the structural mode, called coincidence phenomenon. It is

associated with an increase at the vibration response in the this frequency. This phenomenon is usually observed at frequencies close to the cut-on frequencies of the cylindrical shells due to the circumferentially combined character of the structural and acoustic waves of the wavenumber (BULL; NORTON, 1980). Although this is an important phenomenon of coupling between these two domains (FAHY; GARDONIO, 2015), there are no studies on two-phase internal flow applications.

The acoustics of fluid-filled pipes is not a new topic. For instance, Fuller and Fahy (FULLER; FAHY, 1982) investigated the dynamic behaviour of thin-walled cylindrical elastic shells filled with fluid, showing that energy in specific modes can be transferred between the fluid and the structure depending on the excitation frequency and the modal characteristics of the coupled system. Norton and Bull (NORTON; BULL, 1984) investigated the phenomenon of acoustic radiation in turbulent internal flow, where identified the coupling of structural and acoustic modes close to the cut-on frequency. More recently, Kirby and Duan (KIRBY; DUAN, 2019) demonstrated the phenomenon of energy transfer between the fluid and the structure wave modes using a semi-analytic method to couple the elastodynamic wave equation for the pipe wall to the convected wave equation for sound propagation in a uniform fluid flow. They have shown that the fundamental fluid type mode comes close to a structural type mode, but veer apart and, consequently, mechanical energy is transferred from the fluid to the structure due to this type of wave coupling (MANCONI; MACE, 2017). Thus the mean flow in cylindrical shell significantly changes its sound radiation characteristic. Maxit and Karimi (MAXIT; KARIMI; GUASCH, 2021) show that even a weakly spatially correlated Turbulent Boundary Layer (TBL) in the interior of a pipe yields strongly spatially correlated vibratory field on the pipe surface, mostly at the cut-on frequencies of the circumferential orders. Carvalho et al. (CARVALHO; FIGUEIREDO; SERPA, 2019), experimentally observes a vibration amplification at high frequencies. Although no physical interpretation is presented, this is used to identify two-phase flow patterns inside the pipe. Remarkably, there is a gap in the literature about the vibroacoustics of pipes conveying two-phase flow.

In this study, the dynamic behavior of a horizontal pipe conveying gas-liquid two-phase is investigated aiming to identify and characterize two-phase flow patterns based on indirect approaches. Firstly, an experimental investigation of the two-phase fluid-structure coupling is proposed for horizontal pipes conveying intermittent flow. Focus is given at frequency bands around the cut-on frequencies of the circumferential wave modes of the pipe. The pipe excitation and response are investigated from pressure and acceleration measurements, as well as a high-velocity camera and a conductance-based void fraction meter array. The images are then post-processed for identification and measurement of the dispersed bubble diameters and the void fraction measurements are used for synchronisation of the sensor measurements. Two experimental conditions with intermittent flow are investigated. It is shown that the pipe vibration response is significantly amplified at the circumferential wave modes cut-on frequencies and that it is

time-modulated by the passage of the intermittent flow. Consequently, features from the two-phase flow, such as the flow pattern, Taylor bubble velocity and frequency of passage of the bubble, can be estimated from the demodulation of vibration response filtered at the cut-on modes. Posteriorly, a nonlinear analysis of dynamic systems is conducted using techniques such as state space reconstruction, Lyapunov exponent, and correlation dimension with the aim of characterizing the dynamics of intermittent flow. Finally, a methodology for identifying flow patterns is proposed using pressure fluctuations in the time domain signals and nonlinear series analysis techniques. For this purpose, dimensionless parameters such as correlation dimensions, Lyapunov exponents, and Hurst exponents are employed. It is demonstrated that these parameters create a multi-dimensional map with clusters corresponding to different flow patterns.

1.2 Objectives

The overall objective of this study is to propose methodologies for identifying and characterising two-phase flow in horizontal pipes through indirect measurement approaches using non-invasive sensors, such as piezoelectric pressure and vibration sensors. The specific objectives are:

- To develop a methodology for identification the flow pattern based on piezoelectric pressure sensors and structural vibration.
- To develop a methodology for estimating two-phase flow parameters with focus in intermittent flow patterns and based on piezoelectric pressure sensors and structural vibration.

1.3 Thesis outline

The thesis is organised into seven chapters. The current introductory chapter provides the research motivation and objectives. Chapter 2 reviews some fundamentals of two-phase flow in horizontal pipes. Different flow patterns and maps are reviewed along with some classical models to estimate main features of intermittent flow and dispersed bubbles.

Chapter 3 reviews some fundamentals of two-phase flow induced vibration in terms of Euler-Bernoulli beam theory, for low frequency dynamics, and in terms of wave propagation under the cylindrical shell theory, for higher frequencies.

Chapter 4 presents the fundamentals of the signal processing approaches focusing in identification and characterisation of two-phase flow. Also, classical estimators of frequency response function of linear systems is reviewed. Finally, an identification approaches for

nonlinear systems are reviewed in terms of reconstruction of phase space from a single point measurement.

Chapter 5 presents the experimental setup for two-phase flow. The water-air test matrix is also presented in terms of superficial velocities of each phase along with each corresponding flow pattern and initial estimates of the two-phase flow features from intrusive sensors.

In Chapter 6, the experimental results and discussion are presented. An introduction analysis is presented in both time and frequency domain, with a particular focus on the acceleration measurements. Then, the fluid-structure coupling is investigated in terms of the excitation given by the bubble size distribution in the flow, and fluid-structure coupling given around the cut-on frequencies of circumferential wave modes. In this frequency band, it is shown that there is a significant vibration amplification and that some features from the intermittent flow can be estimated from the acceleration signal filtered in this band. The bubble size distribution is estimated by a deep learning model developed for image segmentation tasks. Then, the corresponding frequency spectra is estimated for structural excitation along with the time-domain modulation of bubble distribution. In addition, a FRF-based identification approach is proposed to show the localised nature of the coupling in at the cut-on frequency band. The result is used to propose slug flow characterisation approaches. Then, the results non-linear identification approach is shown in terms of acceleration, pressure and pressure gradient. An investigation show that these quantities are closely related to the void fraction dynamics. The results and analysis are then used to propose flow pattern classification approaches based on a single point measurement from pressure or acceleration sensor.

Conclusions and recommendation for future work are finally presented in Chapter 7.

1.4 Publications and patent submission

As a direct product of this thesis,

- **Das Neves, D.A.**; Vieira, S.C.; Cenzi, J.R.; Barbosa, B.R.; Foresti, B.P.; Castro, M.S. Patent: Innovation Privilege. Registration number: BR102023019733. "Sistema de medição de características de escoamentos multifásicos, método de demodulação para um sistema de medição de características de escoamentos multifásicos e circuitos eletrônico". Registration institution: INPI - National Institute of Industrial Property. Deposit: 30/09/2023.
- **Das Neves, D.A.**; Vieira, S.C.; Cenzi, J.R.; De Paula, A.S.; Castro, M.S.; Fabro, A.T. "Identification of the flow pattern from the experimental pressure signal in horizontal pipes carrying two-phase flows". Experimental Thermal and Fluid Science. Status: under review.

- **Das Neves, D.A.**; Vieira, S.C.; Cenzi, J.R.; Barbosa, B.R.; Dos Santo, J.N.; Castro, M.S.; Fabro, A.T. "Development and dynamic characterisation of a conductance-based void meter array", Measurements. Status: under review.
- **Das Neves, D.A.**; Vieira, S.C.; Cenzi, J.R.; Castro, M.S.; Fabro, A.T. "An experimental investigation on the fluid-structure coupling in horizontal pipes conveying two-phase intermittent flow". Applied Acoustics. Status: under review.

As an indirect product of this thesis,

- Matos, P. H.; Vieira, S.C.; **Das Neves, D.A.**; Fabro, A.T.; Der Geest, C.V.; Iceri, D.M.; Foresti, B.P; Ribeiro, M.P.; Castro, M.S. Patent: Innovation Privilege. Registration number: BR 10 2021 024428 3. "Method and system for measuring the characteristics of a multiphase flow". Registration institution: INPI - National Institute of Industrial Property. Deposit: 08/06/2023.
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- **Das Neves, D.A.**; Vieira, S.C.; Cenzi, J.R.; De Paula, A.S.; Castro, M.S.; Fabro, A.T. "An experimental investigation on the two-phase intermittent flow coupling with structural vibration of horizontal pipes ". COBEM - 27th International congress of mechanical engineering. Florianópolis-Brazil, 2023.

- **Das Neves, D.A.**; Vieira, S.C.; Cenzi, J.R.; De Paula, A.S.; Castro, M.S.; Fabro, A.T. "Identification of the flow pattern from the experimental pressure signal in horizontal pipes carrying two-phase flows". COBEM - 27th International congress of mechanical engineering. Florianópolis-Brazil, 2023.
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2 Fundamentals of two-phase flow in horizontal pipes

This chapter presents a description of the main concepts of two-phase flow in horizontal pipes. Firstly, the main concepts and fundamentals of two-phase flow are presented, in Section 2. In Sections 2.3 and 2.4, the different flow patterns and maps are briefly reviewed. Afterwards, Section 2.5 presents the characteristics of the intermittent flow as well as some classical models to estimate the slug frequency, translational velocity and liquid holdup. Finally, in Section 2.6, typical models and of oscillating bubbles, such as diameters and frequency of oscillation are briefly reviewed.

2.1 Two-phase flows in horizontal pipes

Two-phase flows are characterized by the presence of interfaces between the phases and discontinuities in the properties associated with flow patterns. In pipes conveying gas-liquid flow, common flow patterns are classified as stratified, annular, dispersed bubble and intermittent flow. The later is typically the most relevant for industrial purposes as it can create excessive levels of vibrations which may lead to structural pipe failures (MIWA; MORI; HIBIKI, 2015). An intermittent flow can be modelled as unit cell varying from an elongated air bubble with a liquid film in segregated (stratified) flow pattern to a liquid slug with or without dispersed gas bubbles (TAITEL; BARNEA, 1990; FABRE; LINÉ, 1992; NETTO; FABRE; PERESSON, 1999), both connected by a turbulent recirculating zone (ISHII; HIBIKI, 2011). Although much research effort has been made in last decades, modelling gas-liquid flows still depends on several experimental data and closure laws (WU et al., 2017).

In the flow patterns of dispersed bubbles and slug, the distribution of dispersed small bubbles plays a crucial role, directly influencing the system's behavior. Parameters such as diameter, volume, interfacial area, and numerical density of bubbles become fundamental to understand and analyze these transient flows. In the context of dispersed bubbles, where bubbles are uniformly distributed in the liquid phase, while in slug patterns, bubbles are grouped in the liquid slug and mixing region. Investigating the dynamics of these bubbles involves the study of the phenomena of coalescence, breakup, and two-phase

turbulence, which is beyond the scope of this work (PAOLINELLI; RASHEDI; YAO, 2018; LIAO; LUCAS, 2009b). Another important phenomenon investigated in this work is the Rayleigh-Plesset resonance frequency, which is a crucial phenomenon in the study of interactions between gas bubbles and acoustic fields. The Rayleigh-Plesset model describes the oscillations of spherical gas bubbles immersed in an incompressible liquid, taking into account various parameters such as equilibrium radius, internal and external pressure, liquid density, thermal and liquid viscosities, and surface tension (BRENNEN, 2005). The significance of the Rayleigh-Plesset resonance frequency is highlighted by the system's ability to adjust to the specific frequency of the acoustic field, resulting in a notable amplification of the bubble's response (OMOTESO et al., 2021). This means that when the bubble is excited by a frequency close to its natural resonance frequency, there is a significant increase in the amplitude of its oscillations.

2.2 Fundamentals of two-phase flows

Two-phase flow is characterized by a very complex regime configuration, with a large number of flow variables. According to Ishii e Hibiki (2011) this complexity is due to the existence of a deformable and moving interface and discontinuities of fluids properties close to this interface. In order to evaluate the two-phase internal flow, several physical quantities were established and some of these will be presented according to work by Wallis (1969) and Shoham (2006). Figure 2.1 presents a schematic representation of the relationship between two phases flowing into a pipe, where the gas and liquid phases are separated. This represents the no-slip condition case, in which the gas and the liquid phases travel at the same velocity ($V_G = V_L$). In which V_G and V_L are the actual velocities of the liquid and gas phases, respectively. Note that each of the phases occupies only a part of the cross-sectional total area (A), with A_G and A_L being the cross-sectional areas of the liquid and gas phases. The interface length is S_I and the wetted perimeter of the gas and the liquid phases are S_G and S_L , respectively.

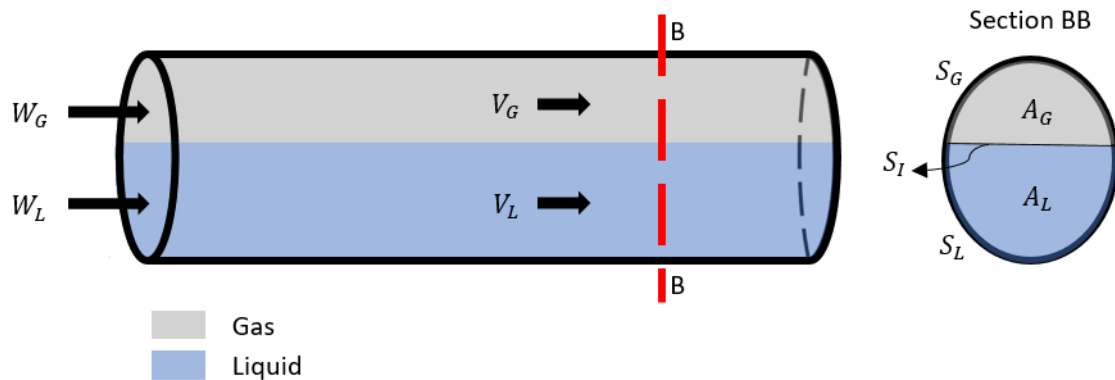


Figura 2.1 – Two-phase flow in a horizontal pipe (on the left) and cross-sectional area (on the right).

The mass flow Flow Rate is a measure that describes the amount of mass of a fluid passing through a specific section in a specific interval of time, expressed in kg/s . The total mass flow rate (W) is the sum of the mass flow rates of the liquid (W_L) and gas phases (W_G). The mass flow rate for each phase is given by,

$$W_L = \rho_L V_L A_L, \quad (2.1)$$

$$W_G = \rho_G V_G A_G. \quad (2.2)$$

where ρ_L and ρ_G are the densities of the liquid and gas phase.

The volume of fluid that passes through a specific section of a system during a specific time interval is referred to as the volumetric flow rate, expressed in units of m^3/s . The total volumetric flow (q) can be defined with the sum of liquid volumetric flow rate (q_L) and gas volumetric flow rate (q_G),

$$q = q_L + q_G. \quad (2.3)$$

A fundamental dimensionless variable to define the two-phase flow is the *in situ* volumetric fraction of the phases, where the volumetric fraction of the liquid phase is also known as holdup (H_L) while the volumetric fraction of gas phase is defined by void fraction (α). In single-phase flow, α and H_L take extreme values, being zero when there is no presence of the gas phase and one when the liquid phase is absent. In two-phase flow, $0 < H_L, \alpha < 1$ and $H_L + \alpha = 1$ reflects the complementary nature of the liquid and gas phases. These conditions ensure that the entire volumetric of the flow element is occupied by one of these phases, without overlap. The relationship between the H_L and the α with the occupied areas is given by,

$$H_L = \frac{A_L}{A}, \quad (2.4)$$

$$\alpha = \frac{A_G}{A}. \quad (2.5)$$

The superficial velocity of a phase is characterized by the volumetric flow rate of the respective phase, denoting the volumetric flow rate per unit area. In simpler terms, it represents the velocity that would manifest if only that specific phase were in motion within the pipe. Consequently the mixing velocity (J_M) is the velocity of the central volume and is given by the sum of the superficial velocities of the liquid (J_{SL}) and gas phases (J_{SG}). Thus, superficial velocities are given by the following expressions,

$$J_{SL} = \frac{q_L}{A}, \quad (2.6)$$

$$J_{SG} = \frac{q_G}{A}. \quad (2.7)$$

$$J_M = \frac{q_L + q_G}{A} = J_{SL} + J_{SG}. \quad (2.8)$$

The mean density of the two-phase flow (ρ_M) and its viscosity (μ_M) are given by,

$$\rho_M = \rho_L H_L + \rho_G (1 - H_L), \quad (2.9)$$

$$\mu_M = \mu_L H_L + \mu_G (1 - H_L), \quad (2.10)$$

where μ_L and μ_G are the viscosity of the liquid and gas phases.

The no-slip condition occurs in situations such as homogeneous flow or dispersed bubble flow, where there are high liquid flow rates and low gas flow rates. However, in intermittent flows, the gas and liquid do not move at the same velocity, resulting in slippage between the phases. Consequently, the cross-sectional area of the gas phase decreases, while that of the liquid phase increases (SHOHAM, 2006).

Another important concept in two-phase flow in pipelines is the pressure gradient, which is a measure of the pressure variation per unit length along the pipe. This relationship is mathematically expressed by the equation,

$$\frac{dp}{dL} = -\frac{p_{re2} - p_{re1}}{dL}, \quad (2.11)$$

where dL is the variation in length between these two points, and p_{re2} and p_{re1} represent the pressures at two different points along the length of the pipe.

The total pressure gradient consists of the frictional, gravitational and accelerational pressure gradient components, respectively given by,

$$\frac{dp}{dL} = -\left(\frac{dp}{dL}\right)_{Fric} - \left(\frac{dp}{dL}\right)_{Grav} - \left(\frac{dp}{dL}\right)_{Acc}. \quad (2.12)$$

where frictional, gravitational and accelerational pressure are defined as,

$$\left(\frac{dp}{dL}\right)_{Fric} = \frac{2}{d} f_F \rho_M V_M^2, \quad (2.13)$$

$$\left(\frac{dp}{dL}\right)_{Acc} = \rho_M V_M \frac{dV_M}{dL}, \quad (2.14)$$

$$\left(\frac{dp}{dL}\right)_{Grav} = \rho_M g \sin(\theta). \quad (2.15)$$

where f_F is the Fanning friction factor and θ is the pipe inclination.

Combining Equations 2.13, 2.14 and 2.15, the total pressure gradient is defined by,

$$\frac{dp}{dL} = \frac{\frac{2}{a} f_F \rho_M V_M^2 + \rho_M g \sin(\theta) + G^2 v_{GL} \frac{dx}{dL} - \frac{G^2}{\rho_M} \frac{1}{A} \frac{dA}{dL}}{1 + G^2 \left(x \frac{dV_G}{dp} + (1 - x) \frac{dV_L}{dp} \right)}, \quad (2.16)$$

where $G = \rho_M V_L$ for no-slip condition, v_{GL} is the relative velocity between phases $v_{GL} = V_G - V_L$ and $x = W_G / (W_G - W_L)$.

In a two-phase flow, the pressure gradient depends on the flow pattern, for example, the pattern of stratified flow, which has low gas and liquid velocities, has a low pressure gradient compared to slug flow patterns (GOVIER; AZIZ, 1972).

2.3 Flow Patterns

The geometric distribution of the components in a two-phase flow is called flow pattern or regime flow. These structures depends on the pipe geometry, phases velocities and physical properties of the phases. The interfacial is defined as discontinuity, since it has neither area nor mass but it can accumulate energy and entropy. Ordinarily, the flow patterns are recognized by visual inspection though other techniques can be found, e.g., analysis of volumetric fraction or standard deviation of pressure fluctuation signals (BRENNEN, 2005). Thereby, the main motivations for identifying flow patterns are in the relationship between the flow pattern with its corresponding pressure drop, liquid holdup and mass transfer coefficients (SHOHAM, 2006). Changes in flow patterns can significantly alter these parameters with potentially catastrophic consequences in piping systems.

In two-phase flow, different flow patterns are observed as a function of the pipe inclination, since the gravitational force exerts a significant influence on the distribution of components in the pipe. Considering the gas-liquid, the flow pattern can be grouped into three categories: separate phases, intermittent and dispersed flow. The separate flow consists of the parallel flow of components bounded by an interface. On the other hand, disperse flow are characterized by small droplets or bubbles from one of the component dispersed in the continuous component. The intermittent flow consists of alternating between the other two patterns in which kinematic waves grow in a nominally stable flow to create a significant phase separation (BRENNEN, 2005). Thus, Figure 2.2 presents the geometries of the different flow patterns. The flow patterns are subdivided in a convention followed throughout this work:

- Stratified-smooth (SS): it occurs at relatively low gas and liquid flow rates and the gas-liquid interface is smooth.

- Stratified-Wavy (SW): at higher gas rates, results in the formation of stable waves at the gas-liquid interface.
- Elongated-Bubble (EB): it occurs for lower gas rates and is formed by a Taylor bubble and a liquid piston, that is free of small bubbles.
- Slug (SL): at higher gas flow rates, the liquid piston has small bubbles in the form of an eddy.
- Annular Flow (AN): it occurs at very high gas flow rates, the liquid-phase flows as a thin film around the pipe wall and the gas-phase in a core of high velocity with small drops of liquid.
- Wavy-Annular (WA): considered a transition pattern between the slug and the annular, the liquid moves forward on the upper surface of the pipe with frothy waves.
- Dispersed-Bubble (DB): at very high liquid flow rates, the two phases has at the same velocity and the flow is considered homogeneous no-slip.

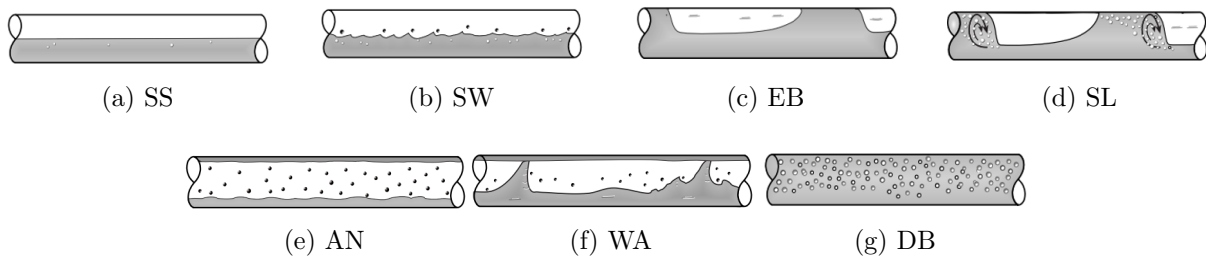


Figura 2.2 – Illustration of flow patterns in horizontal and near horizontal pipes according to [Shoham \(2006\)](#): (a) Stratified-Smooth (SS), (b) Stratified-Wavy (SW), (c) Elongated-Bubble (EB), (d) Slug (SL), (e) Annular (AN), (f) Wavy-Annular (WA) and (g) Dispersed-Bubble (DB).

The intermittent patterns, which attracts the most interest from the industry, consists of alternating the repetitive arrangements called unit cells. Each cell has a typical length and velocity along the pipe, consequently with a characteristic frequency. These cells are composed of liquid pistons and a Taylor bubble. Hence, this patterns have complex hydrodynamics with unsteady flow behaviour characteristics, since it alternates between stratified patterns and dispersed bubbles with complex transition regions.

2.4 Flow Pattern Map

Flow pattern maps are graphic mechanisms in which it is possible to determine the flow pattern as a function of a given flow condition, such as superficial velocities of phases, mass flow rates or the momentum fluxes. The transition between the flow patterns

can be quite unpredictable, since they may depend on characteristics such as roughness of the walls. Therefore, the limits of the flow pattern are not distinct lines, but transition zones. Several authors have proposed flow patterns maps but consider the operational and geometric aspects of the respective systems used, i.e., empirical maps. Accordingly, there are many discrepancies considering the different diameters of the pipes, as well as the different fluids used by each one.

The first map was proposed by Baker (1954) for gas-liquid flow in horizontal pipes, the coordinates were based in mass flow of each phase. This flow map considers variables like specific masses, viscosity and interfacial stresses of the phases. After that, the original map from Baker was modified by Scott (1964) called "Modified Baker's map", trying to include the effect of the pipe diameter. Then, Mandhane, Gregory e Aziz (1974) proposed a flow pattern map based on experimental data from 1962 to 1973 listed in the database of the American Gas Association/American Petroleum Institute (AGA/API). A wide range of pipe diameters (1.3 to 15 cm), flow rates and fluid properties of gas-water flow in the horizontal pipe were analyzed. Thus, this map represents "an average" of all the other maps. Consequently, it has the same previous limitations, which no physical basis was given for choosing the mapping coordinates.

Taitel e Dukler (1976) proposed a flow pattern map with transition criteria considering physical and phenomenological mechanisms in the horizontal and slightly inclined pipe. Figure 2.3 presents the Taitel and Dukler map, where X serves as the parameter introduced by Lockhart e Martinelli (1949), F represents a Froude number that has been adjusted by the density ratio, T can be interpreted as the ratio of turbulent to gravity forces influencing the gas, and K is the product of the modified Froude number and the square root of the superficial Reynolds number of the liquid. This map was developed using theoretical methods and considers important flow parameters, such as the pipe diameter, inclination angle, physical properties of the phases and gas and liquid flow rates. This model is the most widely used flow pattern map for horizontal two-phase flow and it is applicable for steady-state fully developed flows. Although the map demonstrates a good performance for of small diameter and low pressure two-phase flow, large discrepancies are observed in other flow conditions.

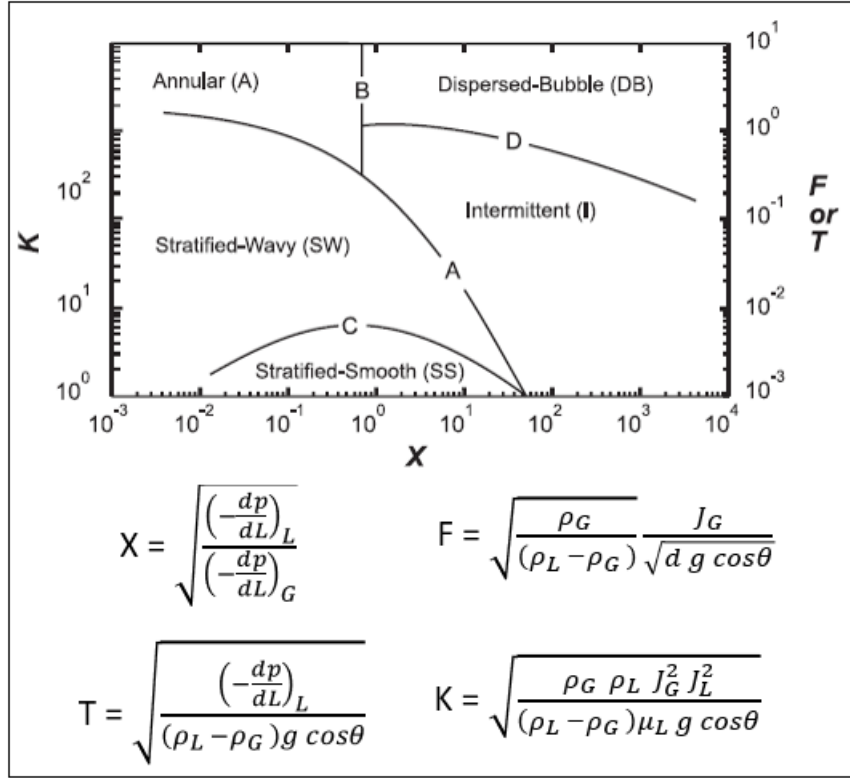


Figura 2.3 – Taitel and Dukler (1976) flow patterns map for horizontal two-phase flow (TAITEL; DUKLER, 1976).

Afterwards, Barnea (1987) developed a mechanistic model based in previous works of the literature, including the previously mentioned work of Taitel e Dukler (1976). It incorporates variables such as flow rates, fluid properties, pipe diameter and angle inclination in the flow pattern transitions. Accordingly, many flow pattern map are found in the literature, and these empirical maps are restricted by the conditions of the experimental. In addition, the pattern near the transition boundary in the flow pattern map is a transition flow, because it is difficult to distinguish the flow pattern in this region.

2.5 Characteristics of the intermittent flow

Figure 2.4 presents the physical model for slug flow. It consists of alternating repetitive arrangements, called unit cells with length L_U along the pipe with a characteristic frequency. These cells are composed of liquid pistons aerated by small dispersed bubbles of length L_{LP} followed by a Taylor bubble of length L_{TB} . In addition, there is a turbulence zone behind the Taylor bubble passage defined as L_M .

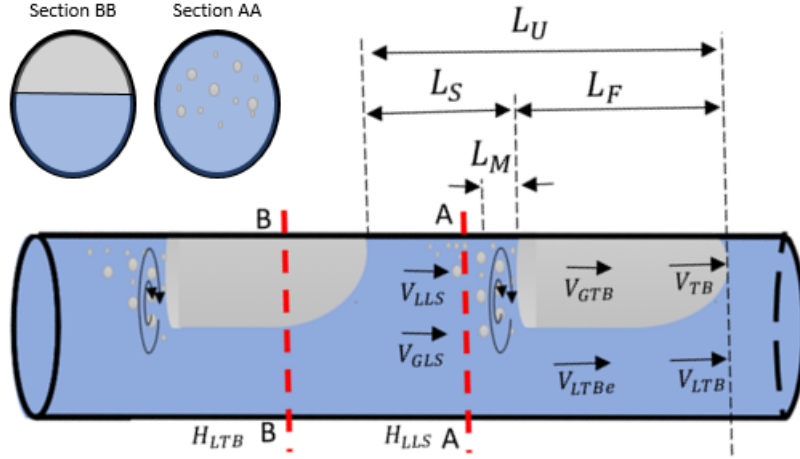


Figura 2.4 – Slug flow characteristics (adapted) (SHOHAM, 2006).

The translational velocity is V_{TB} , the liquid film velocity and Taylor bubble velocity in the stratified region are V_{LTB} and V_{GTB} , respectively. Also, the velocities of the liquid and gas phases in the liquid piston are denoted as V_{LLS} and V_{GLS} , respectively. The liquid holdup in the liquid piston is H_{LLS} . Finally, the liquid holdup in the stratified region is H_{LTB} . The translational velocity is higher than the others. Consequently, the liquid is collected in front of the Taylor bubble and disposed on the liquid piston, so the distribution velocity is given by,

$$V_{TB} > V_{GTB} > V_{GLS} > V_{LLS} > V_{LTB}. \quad (2.17)$$

The intermittent flows can be presented in two different patterns, namely the slug and the elongated bubble. The distinction between these two pattern is given by the existence of gas bubbles inside the liquid piston. Therefore, the intermittent regimes are characterized by specific parameters, such as the slug frequency, translational velocity, liquid holdup, slug and bubble Taylor length. Thus, the main models for estimating some of these parameters will be discussed in the following section.

2.5.1 Slug frequency

Slug frequency is defined as the number of slug passing at a fixed point along a pipe over a certain period of time. Thus, the prediction of the slug frequency is important for the designing of transportation pipelines and estimating the pressure drop in the slug flow. Based on empirical and mechanistic arguments, different methods to predict the slug frequency (f_{slug}) have been proposed by several authors.

Gregory e Scott (1969) experimentally investigates the slug frequency in two-phase flow for horizontal pipe of 19 mm diameter. The system was composed of carbon dioxide

and water. The authors proposed an expression to calculate the frequency given by,

$$f_{slug} = 0.00226 \left[\frac{J_L}{ga} \left(\frac{19.75}{J_M} + J_M \right) \right]^{1.2}. \quad (2.18)$$

where g is gravitational acceleration and a is diameter of the pipe.

Heywood e Richardson (1979) obtained an empirical correlation using the Gamma Ray absorption technique for detecting the phase of two-phase flow given by,

$$f_{slug} = 0.0364 \left[\frac{J_L}{J_M} \left(\frac{2.02}{a} + \frac{J_M^2}{ga} \right) \right]^{1.08}. \quad (2.19)$$

Manolis, Mendes-Tatsis e Hewitt (1995) proposed an experimental correlation for of slug frequency in horizontal pipes. The results was compared with other models and the Gregory e Scott (1969)'s correlation showed reasonable agreement. As a result, the Gregory and Scott's correlation was modified as follows,

$$f_{slug} = 0.0037 \left[\frac{J_L}{ga} \left(\frac{J_L}{ga} + \frac{25 + J_M}{J_M} \right) \right]^{1.8}. \quad (2.20)$$

Schulkes (2011) studied the slug frequencies varying parameters such as pipe diameters, pipe inclination, superficial velocities and liquid viscosities. It was found that, for low viscosity of the liquid phase the slug frequency is defined as,

$$f_{slug} = \frac{J_M}{d} \Psi(\lambda_G) \Phi(Re_L), \Theta(\theta, Fr), \quad (2.21)$$

where,

$$\Psi(\lambda_G) = 0.016\lambda_G(2 + 3\lambda_G), \quad (2.22)$$

$$\Phi(Re_L) = \begin{cases} 12, 1Re_L^{-0.37} & \text{for } Re_L < 4000, \\ 1 & \text{for } Re_L \geq 4000. \end{cases} \quad (2.23)$$

$$\Theta(\theta, Fr) = \begin{cases} 1 + \frac{2}{Fr} \sin(\theta) \sqrt{|\theta|} & \text{for } |\theta| \leq 0.17, \\ \frac{1.8}{Fr} (0.6 + 2\theta - \theta^2) & \text{for } |\theta| > 0.17. \end{cases} \quad (2.24)$$

and λ_G is no-slip void fraction,

$$\lambda_G = \frac{J_G}{J_L + J_G}, \quad (2.25)$$

Re_L is the Reynolds number based on superficial liquid velocity,

$$Re_L = \frac{\rho_L J_L a}{\mu_L}. \quad (2.26)$$

and the Froude number,

$$Fr = \frac{J_L}{\sqrt{ga \cos(\theta)}}. \quad (2.27)$$

[Al-Safran \(2016\)](#) proposed a empirical correlation based in the Poisson probability to estimate the slug frequency in horizontal pipe. A total of 473 data points is used in this study with a wide range of physical fluid properties, geometrical and operational conditions. The correlation is given by,

$$\ln(f_{slug}) = 1.51 - 17.04a + 0.77 \ln(J_L) - 0.181 \ln(S_f), \quad (2.28)$$

where S_f is the slip factor,

$$S_f = 1 + 1.95 \mu_L^{0.01} \frac{J_G^{1.5}}{J_L}. \quad (2.29)$$

In conclusion, several authors has studied the characteristics of the slug flow for a range of fluid properties, pipe diameters and inclination. Thus, the main points observed are that slug frequency is directly proportional to liquid superficial velocity and inversely proportional to pipe diameter. In contrast to gas superficial velocity, which has a weak influence ([WANG; GUO; ZHANG, 2007](#); [AL-SAFRAN, 2009](#)).

2.5.2 Translational velocity

Information about the hydrodynamics of the Taylor bubble is extremely important because it carries most of the gas in the unit cell. Several researchers have performed theoretical and experimental research to obtain correlations of the translational velocity for different flow configurations.

[Gregory e Scott \(1969\)](#) developed a slug frequency correlation, presented in the previous section, and also analyzed the translational velocity (V_{TB}). From the results obtained, it was identified that the translational velocity is directly proportional to the mixture's superficial velocity. The authors proposed an expression to calculate the translational velocity given by,

$$V_{TB} = 1.35 J_M. \quad (2.30)$$

The correlation proposed by the authors neglected the drift velocity, arguing that gravity has no effect on horizontal pipe (GREGORY; SCOTT, 1969). Later on, Nicholson, Aziz e Gregory (1978) developed an expression considering the drift velocity and assumes a constant slug length given by,

$$V_{TB} = 1.128J_M + 0.28. \quad (2.31)$$

Ferré (1992) studied the translational velocity in the low pressure for two-phases flow. The proposed correlation presents two critical Froude numbers,

$$\begin{aligned} \text{if } Fr < 2 : V_{TB} &= 1.1J_M + 0.45\sqrt{ga}, \\ \text{if } Fr < 8 : V_{TB} &= 1.3J_M, \\ \text{if } Fr > 8 : V_{TB} &= 1.02J_M + 3\sqrt{ga}. \end{aligned} \quad (2.32)$$

Bendiksen (1984) studied the correlation between the drift velocity and inclination angle of the pipe. It was observed that the drift velocity in the horizontal case is greater than the vertical case. A critical Froude number was noticed and the translational velocity as given by,

$$\begin{aligned} \text{if } Fr < 3.5 : V_{TB} &= 1.05J_M + 0.342\sqrt{ga}, \\ \text{if } Fr > 3.5 : V_{TB} &= 1.2J_M. \end{aligned} \quad (2.33)$$

2.5.3 Liquid holdup

Gregory, Nicholson e Aziz (1978) proposed a correlation of liquid holdup in horizontal gas-liquid flow. Capacitance sensors have been used for the measurements and the diameter of the pipe was the 2.58 cm and 5.12 cm. It is found that the liquid holdup correlates with the mixture velocity,

$$H_{LLS} = \frac{1}{1 + \left(\frac{J_M}{8.66}\right)^{1.39}}. \quad (2.34)$$

Barnea e Brauner (1985) also developed a model for the prediction of liquid holdup in horizontal gas-liquid flow that assumes a balance between breakage forces due to a turbulence and coalescence due to gravity. The proposed correlation is given by,

$$H_{LLS} = 1 - 0.058 \left[2\sqrt{\frac{0.4\sigma}{(\rho_L - \rho_G)g}} \left(\frac{\rho_L}{\sigma}\right)^{0.6} \left(\frac{2}{a}f_F J_M^3\right)^{0.4} - 0.725 \right]^2, \quad (2.35)$$

where σ is the surface tension and f_F is the Fanning friction factor.

[Marcano et al. \(1998\)](#) studied experimentally the liquid holdup using capacitance sensors in a horizontal pipe of 77.9 mm in diameter, proposing the liquid holdup expression

$$H_{LLS} = \frac{1}{1.001 + 0.0587J_M + 0.0118J_M^2}. \quad (2.36)$$

[Gomez, Shoham e Taitel \(2000\)](#) studied the effect of the angle of inclination in estimation of liquid holdup. The experimental data was obtained from six different studies. Thus, it was noticed that the liquid holdup decreases with the increase in the pipe inclination angle. The proposed correlation is given as

$$H_{LLS} = 1.0e^{-(7.85 \times 10^{-3}\theta + 2.48 \times 10^{-6}, Re_{LS})}, \quad (2.37)$$

where Re_{LS} is the slug Reynolds number,

$$Re_L = \frac{\rho_L J_M a}{\mu_L}. \quad (2.38)$$

The liquid holdup correlation obtained by [Wang et al. \(2014\)](#) is based in the shear between slug front and pipe wall and shear between slug body and liquid film. Then, the mechanistic model uses slug characteristics for calculating the liquid holdup as

$$H_{LLS} = 1 - \frac{C_E[V_{GTB}(S_G + S_I)d_G + (V_{GTB} - V_{LTB})(S_I + S_L)d_L]}{(V_{TB} - V_{GTB})\frac{\pi d^2}{4}}, \quad (2.39)$$

where d_G and d_L is hydraulic diameter of gas and liquid phase, respectively. The dimensionless coefficient C_E , it can be expressed as

$$C_E = 0.019 \ln(Fr) + 0.01. \quad (2.40)$$

In general, the liquid holdup is inversely proportional to the gas superficial velocity and directly proportional to the liquid superficial velocity. In addition, for high viscosity, the liquid slug is laminar because of the low slug Reynolds number.

2.6 Bubbles in Liquid

The distribution of bubbles in the dispersed phase can affect the behaviour of flow systems for different flow pattern. Since the shape and size spectrum of these bubbles can develop in conjunction with continuum phase transport altering the interfacial area that is available for mass transfer between the phases ([YEOH; TU, 2013](#)). Dispersed phase bubbles present in the dispersed bubble patterns or in the liquid piston in the intermittent pattern are deformable and compressible, in two-phase gas-liquid flows. More significantly,

the study of this phenomenon involves understanding the processes of coalescence break-up as well as any phase changes occurring. The coalescence process is responsible for the evolution of fluid bubble sizes, together with break-up and mass transfer due to particle growth. [Liao e Lucas \(2009a\)](#) pointed out a variety of mechanisms for breaking the bubble that include turbulent fluctuation and collision or turbulent shearing, viscous shear force, interfacial instability and shearing-off process.

The spectrum of bubble sizes follows a pattern reminiscent of the Richardson turbulence cascade, where larger bubbles fragment into smaller ones successively. This process continues until reaching the Hinze scale, at which point the influence of surface tension becomes dominant, preventing further fragmentation of bubbles by turbulence ([QI; Mohammad Masuk; NI, 2020](#)). [Zhao et al. \(2021\)](#) investigated the bubble size distribution in a horizontal rectangular pipe. The bubble size spectra indicate a critical length scale, considered to be the Hinze scale, and the spectral shape is related to turbulent fragmentation, thus enabling the calculation of the turbulence dissipation rate. Figure 2.5 presents the variations of bubble size of different orders of magnitude at the three background velocities. The bubbles larger than Hinze diameter ($d_b > d_H$), called super-Hinze region, tend to follow a $-10/3$ power scaling with respect to the major length. In addition, the surface tension can hold the bubble shape against the shear forces in a turbulent flow. However, for a diameter smaller than the diameter of Hinze ($d_b < d_H$), called Hinze-scale, the surface tension can hold the bubble shape against the shear forces in a turbulent flow.

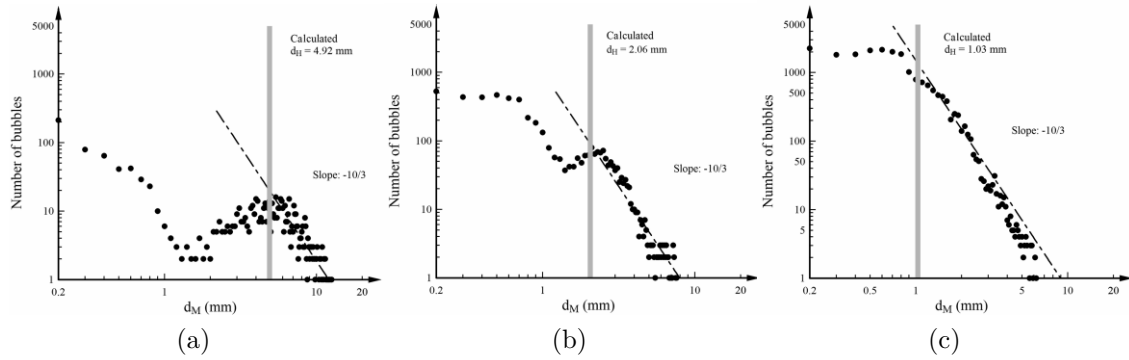


Figure 2.5 – Bubble size spectrum and Hinze scale at different flow velocities: (a) $V_m = 1$ m/s, (b) $V_m = 2$ m/s, (c) $V_m = 3$ m/s ([ZHAO et al., 2021](#)).

Another important phenomenon is the vibrational resonance of gas bubbles immersed in a liquid phase, which is a vibratory phenomenon that occurs when these bubbles are exposed to an acoustic field, characterized by pressure waves in the surrounding liquid medium. This phenomenon is often analyzed in the context of the Rayleigh-Plesset bubble oscillator, a model that describes the dynamics of a gas bubble in an incompressible liquid. In the following sections, the Rayleigh-Plesset model and the calculation of typical analytical diameters will be briefly presented; however, details about this phenomenon are beyond the scope of this work ([BRENNEN, 2005](#); [OMOTESO et al., 2021](#)).

2.6.1 Rayleigh-Plesset equation

The study of bubble oscillatory dynamics presented in this work is based on the analysis of its radial displacement when immersed in an incompressible and isotropic liquid phase subject to excitation of a pressure field. Consider a spherical bubble of radius $(R(t))$ in an infinite domain of liquid with temperature constant (T_∞) and known far-field pressure (p_∞) . It will also be assumed that temperature $(T_B(t))$, and pressure inside the bubble $(p_B(t))$ are uniform, in other words, the contents of the bubble are homogeneous. The phase change from liquid to vapor occurs at the vapor pressure (p_V) which depends only upon the temperature. With these assumptions the Rayleigh-Plesset equation describes the bubble dynamics is given as,

$$\underbrace{\rho_L \left[R \frac{d^2 R}{dt^2} + \frac{3}{2} \left(\frac{dR}{dt} \right)^2 \right]}_{(1)} + \underbrace{\left(\frac{4\mu_L}{R} \frac{dR}{dt} \right)}_{(2)} + \underbrace{\frac{2\sigma}{R}}_{(3)} = \underbrace{p_{G0} \left(\frac{R_0}{R} \right)^{3K}}_{(4)} + \underbrace{p_V(T_\infty) - p_\infty(t)}_{(5)}, \quad (2.41)$$

where σ is the surface tension, K is the polytropic exponent of gas in the bubble, R_0 is the reference radius and p_{G0} partial pressure of the gas inside bubble. The term (1) refers to the inertia generated around an medium bubble oscillations. The term (2) describes the damping of the bubble oscillation due to the viscosity of the liquid phase. The term (3) characterizes the surface tension of bubbles. The term (4) represents the pressure of gas inside the bubble, while (5) describes all pressures acting on the bubble.

2.6.2 Resonant frequency of the bubbles

As previously discussed, a bubble in a liquid can be considered an oscillator due to the elastic behavior of the non-condensable gas that the bubble contains and the inertia of the liquid. Therefore, a natural frequency is associated with bubble oscillatory dynamics in a liquid that can be derived from the Rayleigh-Plesset equation in the absence of thermal effects and liquid compressibility effects. It is assumed an oscillatory excitation pressure, where $\overline{p_\infty}$ is mean pressure value, ω is angular frequency and $\overline{p}$ is the amplitude of a small oscillatory pressure. The pressure oscillation (p_∞) can be represented by,

$$p_\infty(t) = \overline{p_\infty} + \overline{p}e^{j\omega t}. \quad (2.42)$$

At pressure $\overline{p_\infty}$, the bubble assumes the equilibrium size R_e , where $R_e \ll R_0$, thus the linear dynamic response of the bubble will be represented by,

$$R(t) = R_0 + R_e(t)e^{j\omega t}. \quad (2.43)$$

The variations of the bubble radius can be calculated using the Equation 2.41 with Equations 2.42 and 2.43 as,

$$\omega^2 - j\omega \frac{4\nu_L}{R_0^2} + \frac{1}{\rho_L R_0^2} \left(\frac{2\sigma}{R_0} - 3K p_{G0} \right) = \frac{\bar{p}}{\rho_L R_0^2 \varphi}, \quad (2.44)$$

where $p_{G0} = p_\infty - p_V + (2\sigma/R_0)$ is the partial pressure of gas in the bubble. The natural frequency of oscillation of the bubbles, also named Rayleigh-Plesset frequency (f_{RP}), is represented by,

$$f_{RP} = \frac{1}{2\pi} \sqrt{\frac{1}{\rho_L R_0^2} \left(3K(\bar{p}_\infty - p_V) + 2(3K - 1) \frac{\sigma}{R_0} \right)}. \quad (2.45)$$

Bubble resonance refers to the ability of these spherical structures to respond significantly to oscillating forces, particularly when the frequency of the external force approaches the natural oscillation frequency of the bubble. In the case of radial resonance, bubbles undergo oscillations in their radius, alternating between contraction and expansion, in response to pressure variations induced by the acoustic field.

The Rayleigh-Plesset oscillator models this dynamics, treating the bubble as a particle moving in a time-dependent potential. This potential, whether single or double, reflects the temporal variations in pressure exerted by the acoustic field on the bubble. Radial resonance is particularly significant when the frequency of the acoustic field coincides or approaches the natural oscillation frequency of the bubble.

2.6.3 Diameter of bubbles

For horizontal and slightly inclined flow, Taitel e Dukler (1976) proposed a mechanism based on turbulence versus buoyancy force balance. According to the unified model the transition to dispersed bubble occurs as a result of agglomeration of bubbles and/or migration of bubbles due to buoyancy, to the upper part of the pipe, called "creaming". As previously mentioned, the diameter proposed by Hinze (1955) and later modified by Calderbank (1958) and Barnea e Brauner (1985), is based on the concept of turbulent forces overcoming surface tension forces, dispersing the gas phase into small bubbles in the continuous phase. The Hinze diameter is the maximum stable diameter of the dispersed bubbles and is expressed in the following equation,

$$d_H = (0.75 + 4.15\sqrt{\alpha}) \left(\frac{\sigma}{\rho_L} \right)^{0.6} \left(\left| \frac{dp}{dL} \right| \frac{v_M}{\rho_M} \right)^{-0.4}, \quad (2.46)$$

where $\left| \frac{dp}{dL} \right|$ is the pressure gradient.

In contrast, "creaming" mechanism is based on the concept of turbulent forces overcoming buoyancy forces. Turbulent forces tend to disperse the bubbles while to move

the bubble towards the top of the pipe, promoting coalescence. The balance between these forces yields the critical bubble diameter for “creaming” proposed by [Taitel e Dukler \(1976\)](#), given by,

$$d_C = \frac{3}{8} \frac{\rho_L}{(\rho_L - \rho_G)} \frac{f_M v_M^2}{g \cos \theta}. \quad (2.47)$$

[Brodkey \(1967\)](#) proposed a criterion for a bubble diameter where if the resulted bubble size is smaller than the Brodkey bubble diameter, ($d_b < d_{BK}$), bubbles are spherical behaving like solid spheres, with low rates of collision and coalescence. If resulted bubble size is larger than the Brodkey bubble diameter ($d_b > d_{BK}$), the collision and coalescence rates are high the bubbles start to deform and can form a Taylor bubble. But, if turbulent forces are high enough to break the gas into small bubbles, dispersed bubbles will prevail. The correlation for the Brodkey diameter (d_{BK}) is given by,

$$d_{BK} = \sqrt{\frac{0.4\sigma}{(\rho_L - \rho_g)g}}. \quad (2.48)$$

The Hinze diameter is a fundamental parameter representing the upper limit for bubble diameters within a dispersed flow regime. It signifies the threshold beyond which the interactions between bubbles and the surrounding liquid become increasingly influenced by surface tension, impacting the overall flow dynamics. To sustain the dispersed bubble flow pattern, it is imperative that the characteristic Hinze diameter remains smaller than both the Brodkey diameter and the critical diameter. Mathematically, this relationship is expressed as $d_H \leq d_{BK}$ and $d_H \leq d_C$.

2.7 Waves in a flow

Waves are disturbances that propagate in space and time transporting energy without transporting mass. They can be classified according to their nature, being that a mechanical wave propagates in a medium and an electromagnetic wave propagates in vacuums. Besides, the waves can also be classified according to the type of vibration: longitudinal and transverse. In the longitudinal waves, the disturbance moves parallel to the direction of propagation, the main examples are sound waves and compressional elastic waves. In the transverse waves, the disturbance moves perpendicular to the direction of propagation, some examples are shear and bending waves, water and electromagnetic waves. The waves in the two-phase flow are mechanical waves and are formed due to variations in the void fraction and the pressure. Consequently, wave analysis is directly correlated to the flow pattern. For example, in a slug flow, the waves are also mainly caused by transient effects of the flow alternating between liquid piston and Taylor bubble([WHITHAM, 1999](#)).

The compressibility characteristics of gas in a liquid-gas flow induce a notable reduction in the velocity of sound, distinguishing it from single-phase flow. This phenome-

non has been extensively explored by various researchers, including Wallis (1969), Samuel Martin e Padmanabhan (1979), Nguyen, Winter e Greiner (1981), Xu e Gong (2008). In general, the analysis of the velocity of sound differs according to the flow pattern. The bubble dispersed flow were developed from the homogeneous no-slip model, condition in which it is considered that the two phases flow with the same velocity. The stratified flow was based on the separated model and the slug flow in function of the Drift-Flux model. According to Shoham (2006) the velocity of sound of the gas (c_G) and the liquid (c_L) phases, from denominator of the Equation 2.16, are defined as,

$$\frac{1}{c_G^2} = \frac{d\rho_G}{dp} = \rho_G^2 \left(-\frac{dV_G}{dp} \right), \quad (2.49)$$

$$\frac{1}{c_L^2} = \frac{d\rho_L}{dp} = \rho_L^2 \left(-\frac{dV_L}{dp} \right). \quad (2.50)$$

The kinematic wave is also named void fraction wave or concentration wave and the general concept was first introduced by Lighthill e Whitham (1955). The oscillations in the void fraction originate the kinematic waves in the flow, defined by Wallis (1969) as "theory of the waves of continuity", since the velocity of the mass wave is the result of the development of the continuity equation. Kinematic wave is a gradient or discontinuity in the void fraction. Wallis (1969) was one of the first authors to derive a simplified equation for the void fraction wave as,

$$\frac{\partial \alpha}{\partial t} + \frac{\partial (\alpha V_G)}{\partial x} = 0. \quad (2.51)$$

Considering the relative movement between the phases, the flow propagates at a velocity that does not necessarily correspond to the velocity of any of the phases. When the pressure gradients associated with the wave are negligible, their velocity of propagation is governed only by the conservation of the mass, namely velocity of the kinematic wave (BRENNEN, 2005). In the slug pattern, according to the works of Saiz-Jabardo e Bouré (1989) and Fabre, Liné e Gadoin (1995) the void fraction wave is conservative and travels with a constant velocity and close to the translational velocity (V_{TB}). Vieira et al. (2021) correlated the translational velocity with the kinematic velocity (c_p) of flow from the analysis of the cross-PSD phase between the normalized differential pressure signals. It was shown that there is a band of the spectrum where the differential pressure signals are coupled with the kinematic waves of void fraction.

For stratified flow, the waves are classified in kinematic and dynamic according to the Kelvin-Helmholtz stability. This criterion was used by Barnea e Taitel (1993) as a condition for the transition between smooth and wavy stratified flow patterns. For low gas velocities, kinematic waves (stable waves) occur and with increasing gas velocities they become dynamic waves (unstable waves) (WALLIS, 1969; LOPEZ; FULLMER;

[VAIDHEESWARAN, 2013](#)). The velocity of the waves kinematic and dynamics can be correlated to the velocity of the interface between the phases.

For flow in the dispersed bubble pattern, the void fraction waves are attenuated and are directly proportional to the increase in the mean void fraction of the flow ([SONG; NO; CHUNG, 1995](#)). The kinematic wave in dispersed bubble flow is no-dispersive, low frequency and are dampened by the flow of bubbles and this damping decreases with increased void fraction ([MATUSZKIEWICZ; FLAMAND; BOURÉ, 1987](#)).

3 Structural dynamics of pipe

This chapter reviews the main studies of two-phase Flow Induced Vibration (FIV) and Fluid–Structure Interaction (FSI), highlighting important points of each study that contribute to the analysis of the results of this work. Then, in Section 3.2, the equations of motion of the pipe for single-phase and two-phase flow are briefly presented. Afterward, in Section 3.3, the structural dynamics analysis is described based on the Euler-Bernoulli beam theory and cylindrical shell theory.

3.1 Fluid–structure interaction

Several researchers have been engaged in investigating the vibration behaviour of structural systems involving cylindrical pipelines designed for fluid transport, e.g. [Paidoussis \(1998\)](#), [Ortiz-Vidal, Mureithi e Rodriguez \(2017\)](#), [Miwa, Mori e Hibiki \(2015\)](#). These analyses are often conducted numerically or computationally, employing shell or beam models ([ZHANG; LIU; LAM, 2001a](#); [MATOS et al., 2022a](#)). A commonly used approach is the Euler-Bernoulli beam theory, which provides a simplified model assuming that the fluid can be treated as additional mass to the structure. This model is particularly accurate in predicting fundamental frequencies. On the other hand, when considering cylindrical shells, the approach assumes that the thickness is significantly small compared to other relevant dimensions. Furthermore, pipeline vibration due to internal two-phase flow has corresponding characteristic excitation forces for different flow patterns and operating conditions ([KANEKO et al., 2014](#); [ORTIZ-VIDAL; MUREITHI; RODRIGUEZ, 2017](#)). According to [Blevins \(1990\)](#) the Flow Induced Vibration (FIV) is the result of a coupling between hydrodynamic and structural forces. The process begins when the hydrodynamic force acts on the structure and it reacts to produce an opposite force. Thus, the models are separated into structural and fluid domains and coupled with terms of hydrodynamic force. The structural dynamic models, in most cases, are considered linear or quasi-linear systems. On the other hand, the flow dynamics models are more complex due to complex interactions between phases and non-linear behaviours.

The fluid-structure interaction in pipeline systems is an essential field of investigation, where fluid dynamics and structural responses are intrinsically interconnected. However, significant challenges are encountered when dealing with fluid-structure inte-

reaction, encompassing complexity in formulating hydrodynamic forces and predicting structural responses, especially in scenarios characterized by flow fluctuations and external disturbances. [Paidoussis e Li \(1993\)](#) reviews the field of fluid-structure interaction in single-phase fluid-carrying pipes and provides a selective overview of research conducted on the subject. [Firouz-Abadi, Noorian e Haddadpour \(2010\)](#) presents a boundary element model used to analyze the fluid-structure coupling in flexible shells under single-phase flow. [Zhang, Liu e Lam \(2001b\)](#) conducted a coupled structural-acoustic analysis of finite cylindrical shells filled with fluid using the wave propagation approach. A comparison between the coupled and uncoupled analyses was made, using analytical models in the literature and numerical models, respectively. It is demonstrated that the effect of the fluid on the shell is significant, with the coupled frequency being reduced to half of its uncoupled frequency for the shell. [Kirby e Duan \(2019\)](#) observed the phenomenon of energy transfer between the fluid and the structure using a semi-analytic method to couple the elastodynamic wave equation for the pipe wall to convected wave equation for sound propagation in a uniform fluid flow. [Mikami et al. \(2023\)](#) investigated coupled waves between the cylinder walls and an internal liquid. They demonstrated that it is possible to accurately predict coupled wave frequencies using dispersion relations derived from the cylindrical shell theory and the wave equation, incorporating the added mass of the fluid. Additionally, the results indicated the occurrence of coupled waves without external vibration sources and highlighted the variation of the most prominent vibration modes in relation to the flow rate of the internal liquid.

3.2 Equations of motion

Figure [3.1](#) presents a schematic representation of the free body diagram of an infinitesimal pipe-fluid element (ds). The system consists of a uniform pipe conveying fluid with pressure $p(x, t)$ and the reaction to inertial load is $f(x, t)$, imposed on the structure. The pressure gradient in the fluid along the pipe is opposed to the shear stress (τ_w) on the inner wall. Thus, the pipe can be exposed to the shear and normal forces due to flow, such as the longitudinal tension $T(x, t)$ and bending moment $M(x, t)$.

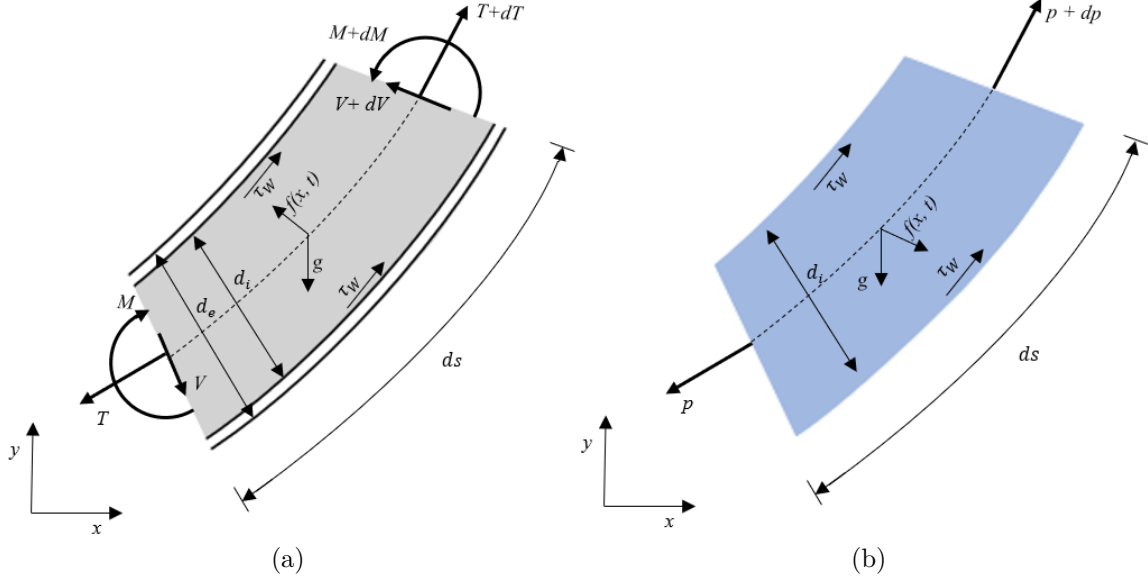


Figura 3.1 – Forces and moments acting on elements of (a) the pipe and (b) the fluid (adapted) (PAIDOUSSIS, 1998).

The equation of motion of the pipe can be derived by considering an infinitesimal pipe-fluid differential section, Figure 3.1, under the assumptions of Euler-Bernoulli beam theory, following the assumptions given by Paidoussis (1998) and Monette e Pettigrew (2004). Therefore, the equation of motion of the pipe radial displacement subjected to a single-phase flow is,

$$\underbrace{EI \frac{\partial^4 w(x, t)}{\partial x^4}}_{(1)} + \underbrace{\rho_F A_F U_F^2 \frac{\partial^2 w(x, t)}{\partial x^2}}_{(2)} + \underbrace{2\rho_F A_F U_F \frac{\partial^2 w(x, t)}{\partial x \partial t}}_{(3)} + \underbrace{(\rho_p A_P + \rho_F A_F) \frac{\partial^2 w(x, t)}{\partial t^2}}_{(4)} = 0, \quad (3.1)$$

where $w(x, t)$ is the radial displacement of the pipe at the position x along the pipe, EI is the flexural stiffness of the pipe, U_F is the flow velocity, A_P and A_F are the cross-sectional areas of the pipe and fluid, ρ_p and ρ_F are the specific mass of the pipe material and the fluid. The term (1) is the pipe flexural elastic force, the term (2) is the fluid centrifugal force due to pipe curvature, the term (3) is the flow Coriolis force due to pipe rotation and the term (4) contains both the fluid and the pipe inertial forces.

For two-phase flow, different dynamic models for the pipe have been proposed in the literature for several flow patterns (SEMLER; LI; PAIDOUSSIS, 1994). For the slug

flow pattern, the equation of motion obtained by [Hara \(1973\)](#) is given by,

$$EI \frac{\partial^4 w(x, t)}{\partial x^4} + \rho_F A_F U_F^2 \frac{\partial^2 w(x, t)}{\partial x^2} + 2\rho_F A_F U_F \frac{\partial^2 w(x, t)}{\partial x \partial t} + (\rho_p A_P + \rho_F A_F) \frac{\partial^2 w(x, t)}{\partial t^2} + \underbrace{A_F \left(\frac{\partial \rho(x, t)}{\partial t} + U_F \frac{\partial \rho(x, t)}{\partial x} \right)}_{(5)} \underbrace{\left(\frac{\partial w(x, t)}{\partial t} + U_F \frac{\partial w(x, t)}{\partial x} \right)}_{(6)} = 0, \quad (3.2)$$

It can be seen that Equation 3.2 is similar to Equation 3.1, the only difference is the last term on the right. The term (5) refers to the variation of the specific mass of the fluid and the term (6) is the variation of moment due to changes in the mass distribution of the fluid.

3.3 Pipe vibration

In this section, two methodologies for characterizing the structural response of the pipe without flow are presented. The first model consists of an Euler-Bernoulli beam with torsional and linear spring boundary conditions, important for characterizing at low frequency. The second approach, on the other hand, analyzes the resonant circumferential modes of the structure at high frequency. Thus, the general response of a piping system is predominantly determined by the relation between the higher-order acoustic modes in the fluid and resonant flexural modes of the pipe wall.

3.3.1 Euler-Bernoulli beam theory

In various contexts involving long and circular cylindrical shell with considerably thick walls, the practical focus is on the mode of propagation characterized by transverse bending, resembling the typical behavior of a beam. In this scenario, the negligibility of transverse section distortion becomes a relevant point of interest. This phenomenon is particularly highlighted, for instance, in the analysis of low-frequency vibration in pipes within industrial environments, such as petrochemical facilities. Therefore, in this specific section, the vibrational response of the duct will be addressed using a model based on the Euler-Bernoulli beam theory. The vibrational response of the pipe is modeled as an Euler-Bernoulli beam. The beam ends are elastically restrained with rotational springs and lateral springs as shown in Figure 3.2. The stiffness of the torsion springs are $k_{\theta 1}$ and $k_{\theta 2}$, while the linear springs are k_1 and k_2 . To determine the differential equation for the lateral vibration of the beam, consider the free body diagram of the Figure 3.2, where $M(x, t)$ is the bending moment, $V(x, t)$ is the shear force and $f(x, t)$ represents the loading per unit of the beam.

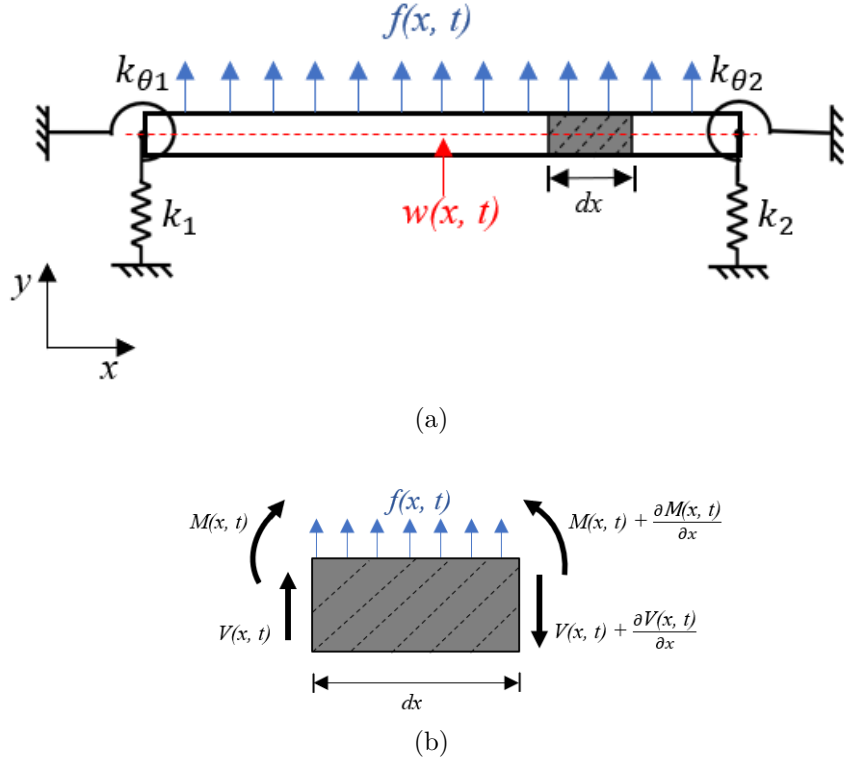


Figure 3.2 – Transverse beam displacements with torsional and linear spring boundary conditions and free body diagram.

The equation of motion of the force in the y direction,

$$-\frac{\partial V(x, t)}{\partial x} + f(x, t) = \rho_p A(x) \frac{\partial^2 w(x, t)}{\partial x^2}, \quad (3.3)$$

where ρ_p is the mass density and $A(x)$ is the cross-sectional area of the beam. The equation of motion of the momentum in the z direction,

$$V(x, t) = \frac{\partial M(x, t)}{\partial x}. \quad (3.4)$$

From the Equations 3.4 and 3.3,

$$-\frac{\partial^2 M(x, t)}{\partial x^2} + f(x, t) = \rho_p A(x) \frac{\partial^2 w(x, t)}{\partial x^2}. \quad (3.5)$$

From the Euler-Bernoulli theory, the relationship between bending moment and the deflection can be expressed by,

$$M(x, t) = EI(x) \frac{\partial^2 w(x, t)}{\partial x^2}, \quad (3.6)$$

where E is the Young module and I is the moment of inertia of the cross section of the beam. From Equations 3.6 and 3.5 is obtained the equation of motion for the forced lateral

vibration of a beam,

$$EI \frac{\partial^4 w(x, t)}{\partial x^4} + \rho_p A \frac{\partial^2 w(x, t)}{\partial t^2} = f(x, t). \quad (3.7)$$

Equation of motion can be solved by using the methods of separation of variables and eigenfunctions expansion,

$$w(x, t) = W(x)T(t). \quad (3.8)$$

Therefore, the Equation 3.7 is rewritten as,

$$\frac{c_b^2}{W(x)} \frac{d^4 W(x)}{dx^4} = \frac{1}{T(t)} \frac{d^2 T(t)}{dt^2} = \omega^2, \quad (3.9)$$

where $c_b = \sqrt{EI/\rho_p A}$ represents the propagation velocity of the wave in the beam and ω is the natural frequency. Note that Equation 3.9 can be separated into two parts in terms of time and position,

$$\frac{d^4 W(x)}{dx^4} - \beta^4 W(x), \quad (3.10)$$

$$\frac{d^2 T(t)}{dt^2} + \omega^2 T(t) = 0, \quad (3.11)$$

where $\beta = \sqrt{\omega/c_b}$. Thus, the beam's natural vibration frequencies are calculated as,

$$\omega = \beta^2 \sqrt{\frac{EI}{\rho_p A}}. \quad (3.12)$$

The solution of Equation 3.7 is expressed by,

$$W(x) = C_1 \cos \beta x + C_2 \sin \beta x + C_3 \cosh \beta x + C_4 \sinh \beta x, \quad (3.13)$$

where C_1, C_2, C_3 and C_4 are constant, as well as β are determined by boundary conditions of the beam. Thus, the linear and torsional spring boundary conditions are, respectively,

$$k_1 w(x, t) = -EI \frac{\partial^3 w(x, t)}{\partial x^3}, \quad k_{\theta 1} \frac{\partial w(x, t)}{\partial x} = EI \frac{\partial^2 w(x, t)}{\partial x^2}. \quad (3.14)$$

$$k_2 w(x, t) = -EI \frac{\partial^3 w(x, t)}{\partial x^3}, \quad k_{\theta 2} \frac{\partial w(x, t)}{\partial x} = EI \frac{\partial^2 w(x, t)}{\partial x^2}. \quad (3.15)$$

By replacing the boundary conditions of Equations 3.16 and 3.15 in Equation 3.13, a transcendental equation is obtained that needs to be solved numerically.

3.3.2 Cylindrical shells

In circular cylindrical shells, the propagation of waves that incorporate distortion in the cross-sectional area becomes practically relevant when the ratio between the cylinder diameter and wall thickness is significant. In the scenario of a wall with uniform thickness and constant mass per unit area, and under the condition that the shell is truly circular in its cross-sectional shape, the acceptable spatial form for the distortion of a cross-section must exhibit periodicity along the length of the circumference. Over the years, numerous theories and approaches have been formulated for the analysis of shell structures. The theories of Love (LOVE, 1944), Flügge (FLUGGE, 1960), Timoshenko (TIMOSHENKO, 1921), Donnell (DONNELL, 1935), Reissner (REISSNER, 1941), among others, have extensively documented the behaviour of shells. Observe that in Figure 3.3, a circular cylindrical shell is illustrated, where the axial, tangential, and radial displacements are denoted by u , v , and w , respectively. In cylindrical shells, unlike plates, flexural waves necessitate consideration of all three displacements u , v , and w due to the curvature of the cylinder wall. This curvature allows "membrane" stresses from direct deformations to induce linearly related radial force components. Radial displacement also generates membrane stresses producing linearly related tangential force components. The magnitude of these coupling forces diminishes with increasing cylinder radius, eventually vanishing in a flat plate with an infinite radius.

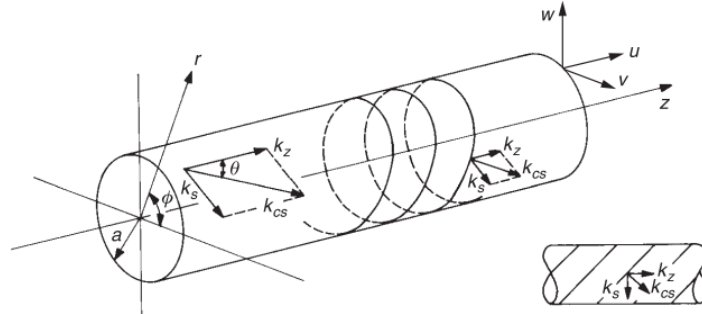


Figura 3.3 – Coordinates, displacements, wavenumber vectors, and helical wavefronts of a circular cylindrical shell (FAHY; GARDONIO, 2015).

As shown in Figure 3.3, structural waves propagate in a helical pattern and can be represented by the wavenumber k_{cs} . This wavenumber is the vector combination of the wave components in the axial (k_z) and circumferential (k_s) directions. Thus, the wavenumber relation is given by,

$$k_{cs}^2 = k_z^2 + k_s^2. \quad (3.16)$$

Figure 3.4 illustrates the modes that describe how waves behave as they propagate around the circumference of the structure. Each circumferential mode is associated with an integer, usually denoted as " n ", indicating the number of lobes or nodes along the

circumference. It is observed that, for $n = 0$, the structure radially expands and contracts symmetrically, referred to as the breathing mode. For the case of $n = 1$, referred to as the beam mode, the structure does not undergo deformation in its cross-sectional area; instead, the main characteristic is a bending pattern along the circumferential direction. The term "beam mode" suggests that the waveform is similar to the behavior of a beam, where the primary deformation occurs along the longitudinal direction. From $n=2$, the wave modes called "cut-on" emerge, in which the modal pattern of a structure consists of a set of $2n$ nodal lines along generators equally spaced in the structure.

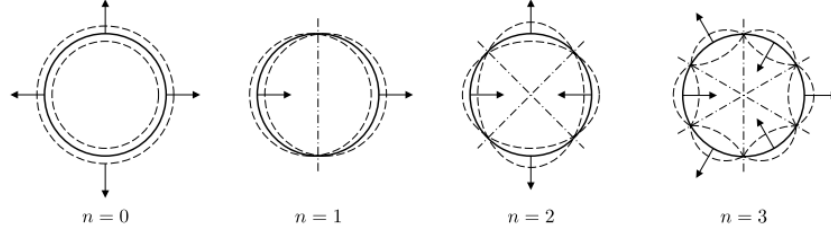


Figura 3.4 – Wave modes of the cylinder in the circumferential direction (Mikami et al., 2023).

The frequency associated with the circumferential mode $n = 0$, "breathing" mode, is called the ring frequency (f_{ring}) and is characterized by having a resonance frequency higher than that for $n = 2$ due to the effect of the curvature of the ring stiffener. Sometimes, the ring frequency can be higher than the frequencies $n = 3, \dots, N$. It is typically used to separate the vibration behaviour into high and low frequency regimes. This frequency is analytically given by,

$$f_{ring} = \frac{1}{2\pi a} \sqrt{\frac{E}{\rho(1-\nu^2)}}. \quad (3.17)$$

For low frequency, domain below the ring frequency is the zone dominated by the stiffness of the structure. In a "bending mode", $n = 1$, the cylindrical shell oscillates around the axial axis while radial displacements remain unchanged, in other words, the structure behaves like a beam corresponds to a rigid body mode. Therefore, in an empty cylindrical shell, the frequencies of the other circumferential modes are determined by the non-dimensional frequency parameter (FAHY; GARDONIO, 2015),

$$\Omega_n^2 = \left(\frac{h}{\sqrt{12}a} \right)^2 n^4 \left[1 - \frac{1}{2} \left(\frac{1}{1-\nu} \right) \left(\frac{4-\nu}{n^2} - \frac{2+\nu}{n^4} \right) \right], \quad (3.18)$$

where $\Omega = \omega/\omega_1$ and $\omega_1 = \sqrt{E/\rho a^2}$.

4 Signal processing and identification approaches

In this chapter, the methods for time series analysis that will be used in the following sections are presented. First, a literature review of the main methods for identifying and characterising two-phase flow patterns is conducted. Subsequently, in Section 4.2 statistical methods are introduced, followed by the discussion of linear analysis methods, Section 4.3, and finally, non-linear analysis methods are explored in Section 4.4.

4.1 Flow pattern identification and characterisation

The identification and characterisation of two-phase flow through indirect methods, based on the analysis of time series, represent an extensive area of study within the scientific community. Matsui (1984) and Matsui (1986b) showed that it is possible to identify patterns of two-phase flows based on the configuration of Probability Density Functions (PDF) of differential pressure fluctuations signals. However, a dependence of the observed slug pattern scales on the distance between the pressure sensor measuring points is reported. Ding et al. (2007b) investigated the relationship between the signal's energy distribution and the flow pattern using the Hilbert-Huang Transform (HHT) on differential pressure signals. It was reported that the extracted energy features provide a good indication of the acoustic state of the two-phase gas-liquid flow. Carvalho, Figueiredo e Serpa (2019) used structural vibration signals from a vertical pipe conducting two-phase flow to classify the flow pattern using techniques such as Root Mean Square (RMS) value and Pearson correlation coefficient. dos Santos Ambrosio et al. (2022) presents a pattern classification approach using the Probability Density Function (PDF) of the estimated void fraction signals from the mixture, which feeds a Support Vector Machine (SVM). Chen et al. (2023) proposes a flow pattern identification method based on Convolutional Neural Networks (CNN), using the HHT and Short-Time Fourier Transform (STFT) of structural vibration signals as input. However, no analysis of the system dynamics is performed.

Despite non-linear dynamics methods being powerful tools for the identification and characterisation of flow patterns, as they seek hidden patterns and underlying structures in data, they still remain largely unexplored. Franca et al. (1991b) investigated

signals of pressure fluctuation in terms of correlation dimension and Hurst exponent and concluded that these coefficients carry information associated with the flow pattern. [Cai, Wambsganss e Jendrzeczyk \(1996\)](#) introduced the application of chaos theory in the context of differential pressure measurement for the identification of water-air flow patterns within a horizontal rectangular channel. The authors utilised the fractal dimension of attractors, Lyapunov exponent, and state space surface analysis to characterise the system's dynamics, demonstrating it to be chaotic. [Wang, Mosdorf e Shoji \(2003\)](#) used differential pressure signals in a two-phase flow at a T-junction to identify bubble, churn, and annular patterns, and the analyses of the Hurst exponent and Lyapunov indicated that, despite being chaotic, they exhibited anti-persistent behaviour. [Jin et al. \(2003\)](#) investigated two-phase water-oil flow in vertical pipelines using conductance signals and demonstrated that the Kolmogorov entropy has a strong correlation with the total flow rate and water-cut. [Fang et al. \(2019\)](#) investigated slug, churn, and bubble patterns using five parameters to quantitatively analyse their respective chaotic attractors: correlation dimension, Kolmogorov entropy, Hurst exponent, mobility, and complexity. Amongst their main results, they show that the embedding dimension was estimated between 4 and 5 using the False Nearest Neighbor method and 6 using Cao algorithm ([ZHANG; LI; FAN, 2020](#)). Kolmogorov entropy increases as the gas volume in the system increases, and the Hurst exponent exhibited anti-persistent behaviour for all patterns analysed. Several researchers have documented the characterisation of non-linear dynamic signals through the application of recurrence analysis ([ZONG et al., 2010](#); [SAINI; BANERJEE, 2021](#); [PAGANO; ANGELI, 2022](#)).

In intermittent flow patterns, essential characteristics for dynamical characterisation are observed, such as void fraction, Taylor bubble velocity, and slug frequency. [Vieira et al. \(2021\)](#) proposed a methodology for in situ estimation of Taylor bubble velocity in slightly inclined annular flow using differential pressure signals. This result was achieved by applying cross-phase Power Spectral Density (PSD) analysis to the differential pressure signals. The cross-correlation function is also used to obtain the delay time between two groups of signals to determine the speed of the Taylor bubble ([CAZAREZ-CANDIA; BENÍTEZ-CENTENO, 2020](#)). [Arabi et al. \(2018\)](#) and [Arabi et al. \(2020\)](#) propose a methodology to estimate the slug frequency by using the dominant frequency obtained through the application of PSD on the pressure drop time signal. When compared to other empirical methods, it was considered the best method for slug frequency measurements. [Matos et al. \(2022c\)](#) proposed a methodology to estimate the void fraction using structural vibration signals in horizontal two-phase flow pipeline systems. The authors utilize the flexural waves in the pipeline under operational conditions to estimate the dispersion curve and correlate it with the density of the two-phase mixture, and consequently, the void fraction.

Precise methods for identifying and characterizing flow patterns play a crucial role in enhancing the safety and efficiency of industrial operations. These approaches, which

encompass time-frequency analysis, statistical analysis, and chaotic dynamics, coupled with a deeper physical understanding, hold the potential to become fundamental tools in industrial operations. Furthermore, this approach can effectively lead to the extraction of relevant features for the application of machine learning techniques.

4.2 Statistical approaches

This section discusses fundamental concepts in statistical analysis, such as: distribution, statistical moments, Pearson and Spearman coefficient, correlation function and power spectral density (PSD). The approach to estimating the delay time using the autocorrelation function and the procedure for calculating the Hurst exponent are also presented.

4.2.1 Probability density function

Probability distribution or Probability Density Function (PDF) describes the relative probability of a continuous random variable taking on a specific value. In other words, it provides information about how the values of a random variable are distributed across its range of possible values (DEVORE, 2012). Consider a continuous random variable then a PDF of X is a function $f(x)$ such that for any two numbers a and b is equal to,

$$P(a \leq X \leq b) = \int_a^b f(x)dx. \quad (4.1)$$

For $f(x)$ to be a legitimate PDF, it must satisfy the following two conditions: the first condition is that $f(x)$ must be greater than or equal to zero, and the second is that the total probability of all possible outcomes must be equal to 1. In mathematical terms, the integral of the joint probability density function over the entire space of variables must be equal to 1. There are many different types of probability distributions, each with its own characteristics. Some of the most common probability distributions are called: uniform, normal (Gaussian), exponential, Poisson, binomial, Bernoulli, Weibull, and log-normal distributions.

The normal distribution, or Gaussian distribution, is indeed one of the most important probability distributions in statistics. It's widely applicable because many real-world phenomena exhibit a distribution that closely approximates the normal curve. Its main characteristics are that the PDF is symmetrical around its mean and unimodal, which means it has a single peak in the mean. In other words, this distribution is entirely determined by the parameters of mean (μ) and standard deviation (σ). The mean sets the centre of the distribution, while the standard deviation regulates the spread of values

around the mean. The Normal distribution is given by the formula,

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}. \quad (4.2)$$

The Lognormal distribution is denominated "lognormal" because the distribution of the logarithms of the values follows a Normal distribution, the variables are strictly positive. The Lognormal Distribution is asymmetric and exhibits an extension towards the right side, characterised by a long positive tail. The following formula describes this distribution,

$$f(x; \mu, \sigma) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma x} e^{-(\ln(x)-\mu)^2/(2\sigma^2)}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases} \quad (4.3)$$

Here, we have only presented a few of the many available probability distributions. Each of them is suitable for modelling various types of phenomena and random variables. The choice of the appropriate distribution is influenced by the nature of the data and the specific characteristics of the problem under analysis.

4.2.2 Moments in statistics

Statistical moments play a vital role in the analysis of data sets and distributions. They serve as tools for understanding and describing the properties and behaviors of the data, aiding in the identification of tendencies, patterns, and distinctive features. Furthermore, moments are essential components in a variety of techniques used in the statistical modelling of phenomena, providing valuable insights for the selection of appropriate models. The most common statistical moment are:

1. The first moment is the mean or expected value, it represents the average value of the continuous variable. Given by the following equation,

$$\mu = \int_{-\infty}^{\infty} x f(x) dx. \quad (4.4)$$

2. The second moment is variance, which measures the dispersion of values in relation to the average. Given by the following equation,

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx. \quad (4.5)$$

3. The third statistical moment is known as asymmetry or skewness, which is a measure that describes the symmetry of the data distribution. It indicates whether the

distribution is skewed to the left (negative) or skewed to the right (positive) relative to the mean. Given by the following equation,

$$\xi_1 = \frac{\int_{-\infty}^{\infty} (x - \mu)^3 f(x) dx}{\sigma^3}. \quad (4.6)$$

4. The fourth statistical moment is called kurtosis and measures the peak or flatness of the data distribution. It provides information about the concentration of values around the mean. Given by the following equation,

$$\xi_2 = \frac{\int_{-\infty}^{\infty} (x - \mu)^4 f(x) dx}{\sigma^4}. \quad (4.7)$$

Additionally, the moments play a significant role in assessing the suitability of the probability distribution adopted to represent the data. In more specific analyses, particularly in the field of time series, these measures also assist in detecting trends and seasonal variations in data sets.

4.2.3 Pearson and Spearman correlation coefficient

In the field of statistics, the Pearson correlation coefficient (ρ) quantifies the linear correlation between two variables, denoted as x and y . Its values range from -1.0 to 1.0, where a value of 1.0 signifies a perfect positive correlation, a value of -1.0 indicates a perfect negative correlation, and a correlation of 0 denotes the absence of a linear relationship between the two variables. The Pearson correlation can be expressed as,

$$\rho = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}, \quad (4.8)$$

where cov is the covariance, σ_x is the standard deviation of x and σ_y is the standard deviation of y .

The Spearman rank-order correlation coefficient is a nonparametric metric used to assess the monotonic relationship between two variables. In cases where there are no duplicated data values, a perfect Spearman correlation of +1 or -1 is observed when each of the variables forms a perfect monotonic function of the other. The Spearman coefficient (ζ) can be expressed as,

$$\zeta = \frac{\text{cov}(R(x), R(y))}{\sigma_{Rx} \sigma_{Ry}}, \quad (4.9)$$

where $\text{cov}(R(x), R(y))$ is the covariance of the rank variables, σ_{Rx} and σ_{Ry} are the standard deviations of the rank variables.

Essentially, both the Pearson coefficient and the Spearman coefficient are correlation analysis tools that investigate the relationship between two variables. However, their

approaches differ and are suited to different data contexts. The Pearson coefficient focuses on assessing the strength and direction of a linear relationship between two continuous variables and is sensitive to outliers, as it is based on measures of central tendency and dispersion. On the other hand, the Spearman coefficient is a nonparametric correlation metric and is centered on evaluating the monotonicity of the relationship between variables, meaning whether one variable increases as the other increases (or decreases as the other decreases), irrespective of the linear or nonlinear nature of the relationship. In summary, the Pearson coefficient quantifies linear relationships, while the Spearman coefficient detects any monotonic relationship, regardless of its linearity.

4.2.4 Correlation function

Autocorrelation Function (ACF) measures the degree of correlation of a time series with itself at different time points, that is, defines the self-similarity of the signal over different delay times (SHIN; HAMMOND, 2008). Then, the ACF (R_{xx}) is defined as the average of a series $x(t)$ multiplied with a delayed version of itself $x(t + \tau)$ given by,

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x(t + \tau)dt. \quad (4.10)$$

On the other hand, cross-correlation measures the degree of correlation between two different time series as one series is shifted in time $y(t + \tau)$ relative to the other. It indicates how one series is related to the other in terms of their temporal shifts. The cross-correlation (R_{xy}) is defined as,

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)y(t + \tau)dt. \quad (4.11)$$

Autocorrelation is commonly used to identify cyclic patterns or seasonality in time series data. In addition, it is possible to observe that the cross-correlation function detects the delay that is associated with the correlated components of the series. Since it finds the components in $y(t)$ that correspond or adjust to $x(t)$.

4.2.5 Power spectral density

The Fourier transform of the signals of the autocorrelation function is defined as Power Spectral Density (PSD). The PSD characterizes the distribution of a signal's power across its various frequency components. This metric provides a detailed analysis of the signal's intensity within each frequency band it contains, enabling the identification of the relative contributions of different frequencies to the overall signal power. It provides the

frequency composition of the signals in terms of the spectral density of its mean-square value (SHIN; HAMMOND, 2008). The auto-PSD function (S_{xx}) is given by,

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx} e^{-j\omega\tau} d\tau. \quad (4.12)$$

Similarly, the Cross Power Spectral Density (CPSD) describes the distribution of cross-power between two time series with respect to the different frequency components contained in these series. The CPSD function (S_{xy}) is defined as,

$$S_{xy}(\omega) = \int_{-\infty}^{\infty} R_{xy} e^{-j\omega\tau} d\tau. \quad (4.13)$$

This relation mathematical relation between the ACF function of a signal and its PSD is known as the Wiener–Khinchin theorem. Additionally, the PSD estimates the power distribution in the domain frequency. In practice, this process occurs by decomposition of averages, where the signal is divided in N overlapping and transformed Fourier windows. Then, the amplitude values at each frequency are averaged in all windows to create a total energy spectrum density.

4.2.6 Time delay estimation

The cross-correlation method is widely used to estimate the time delay between two measurements x and y , spaced at a distance d_x (CARTER, 1987; KNAPP; CARTER, 1976; GAO; BRENNAN; JOSEPH, 2006). These methods can be classified as Basic Cross-Correlation (BCC) and Generalized Cross-Correlation (GCC) depending on the applied weighting function during inverse Fourier Transform. For the BCC this function is simply equal to unity while for the the GCC, different functions can be chosen depending on the characteristics of the problem in order to increase the Signal Noise Ratio (SNR) and sharpen the delay peak in the cross-correlation function (GAO; BRENNAN; JOSEPH, 2006). Considering two wide sense stationary zero-mean signals, x and y , the cross-correlation function is defined by,

$$R_{xy}(\tau_{xy}) = \int_{-\infty}^{\infty} x(t)y(t + \tau_{xy})dt, \quad (4.14)$$

where τ_{xy} is the delay time, which is given by the value that maximizes the cross-correlation function, i.e, the maximum peak value of the function R_{xy} .

From the Wiener-Khinchin theorem, the cross-correlation R_{xy} and the CPSD S_{xy} form a pair of Fourier Transforms (SHIN; HAMMOND, 2008). It provides the frequency composition of the signals in terms of the spectral density of its mean-square value. The

BBC estimator can be obtained as,

$$R_{xy}(\tau_{xy}) = F^{-1}\{S_{xy}\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xy} e^{j\omega\tau_{xy}} d\omega, \quad (4.15)$$

where F^{-1} denotes the inverse Fourier transform.

In the GCC approach, the peak in the ACF associated with the time delay of the input signals can be filtered either in the time domain or in the frequency domain. In the time domain, the signals are filtered before delay, multiplication, and integration, while in the frequency domain, a window or weighting function is applied to the CPSD function before performing the inverse Fourier transform (KNAPP; CARTER, 1976; GAO; BRENNAN; JOSEPH, 2006). Then the GCC-based cross-correlation function R_{xy}^g can be defined as,

$$R_{xy}^g(\tau_{xy}) = F^{-1}\{\Psi_g(\omega)S_{xy}(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi_g(\omega)S_{xy}(\omega)e^{j\omega\tau_{xy}} d\omega, \quad (4.16)$$

where $\Psi_g(\omega)$ is a frequency weighting function.

There are many frequency windowing functions available with filters such as the Phase Transform (PHAT), Wiener filter (Wiener), Smoothed Coherence Transform (SCOT), Maximum Likelihood (ML), and ROTH, with different features that were deeply investigated by Gao, Brennan e Joseph (2006) in the the case of water leak detection in pipes. Moreover, Vieira et al. (2023) used the SCOT frequency windowing for estimating the Taylor bubble velocity from void fraction measurements. The frequency weighting used in this work is the ML estimator that yields the minimum variance of the time delay estimate if the signals are random Gaussian, which is given by

$$\Psi_g(\omega) = \frac{\gamma_{xy}^2}{1 - \gamma_{xy}^2} \frac{1}{|S_{xy}(\omega)|}, \quad (4.17)$$

where γ_{xy} is the frequency dependent ordinary coherence function between x_1 and x_2 . Observe that when $\Psi_g(\omega) = 1$, the GCC reduces to the BCC function defined by Equation 4.15.

4.2.7 Hurst exponent

Introduced by Hurst (1951), the Hurst exponent (h) is a measure for long-term memory of a time series; that is, to measure the amount by which that series deviates from a random walk. In the late 1960s, Mandelbrot e Wallis (1968), Mandelbrot e Wallis (1969) published a series of studies formalising the methodology, called R/S analysis. First, the method considers that a discrete time series $y(n)$ where $n = 1, 2, \dots, N$ is divided into k

non-overlapping segments y_k of size $T = N/k$. Then, the method continues by calculating the mean value of each sub-period y_k ,

$$\bar{y}_k = \frac{1}{T} \sum_{j=1}^T y_k(j), k = 1, 2, \dots, \frac{N}{T}. \quad (4.18)$$

Next, the accumulated departure from the average is calculated for each sub-period y_k ,

$$\chi(i) = \sum_{j=1}^{j=1} (y(j) - \bar{y}_k), i = 1, 2, \dots, T. \quad (4.19)$$

To calculate the (R_k/S_k) associated to the size T , first the ranges R_k are computed for each interval χ_k and then divided by the local standard deviations S_k of each sub-period y_k ,

$$\left(\frac{R_k}{S_k}\right) = \frac{\max(\chi(i)) - \min(\chi(i))}{\sqrt{\frac{1}{T} \sum_{i=1}^T (y(j) - \bar{y}_k)^2}}. \quad (4.20)$$

Then, the mean rescaled range for the corresponding T length is calculated,

$$\left(\frac{\bar{R}}{\bar{S}}\right)_T = \frac{1}{k} \sum_{j=1}^k \left(\frac{R_j}{S_j}\right). \quad (4.21)$$

These calculations are repeated for all possible segment sizes T . The statistics of $(R/S)_T$ follows a power law in relation to the T size,

$$\left(\frac{\bar{R}}{\bar{S}}\right)_T \propto bT^h, \quad (4.22)$$

where b is a constant and h is the Hurst scaling exponent calculated as the slope of the straight line that best fits the $(R/S)_T$ curve on a log-log plot.

The Hurst exponent has values between 0 and 1, and the time series can be classified into three different categories. First, if $H > 0.5$, the time series has persistent increments, i.e, it is likely that the trend pattern of the previous step will be maintained. Second, if $H < 0.5$, the time series has antipersistent increments, i.e., it is likely that the trend pattern of the previous step is reversed, and they represent an oscillating process. Finally, if $H = 0.5$, then each step is independent of the previous ones, and the increment function, which represents the fractional Gaussian noise, corresponds to white noise.

4.3 Identification of linear systems

In this section, it will briefly review the Frequency Response Function (FRF) and its significance in the analysis of linear time-invariant systems. The FRF describes the relation between the input and output of these systems, providing crucial insights into their amplitude and phase characteristics across various frequencies. Additionally, the ordinary coherence, a metric that assesses the linear relationship between signals in the frequency domain, is also presented and it is fundamental for the analysis of experimental measurements.

4.3.1 Frequency response function

Frequency Response Function (FRF) describes the relationship between the output and the input in a linear time-invariant system (CRAIG ROY R.; KURDILA, 2006). Consider a Single-Input-Single-Output (SISO) linear time-invariant system, represented by a unit impulse $h(t)$, where $x(t)$ is the stationary random input being applied to the system, and the stationary response $y(t)$ is given by,

$$y(t) = h(t) * x(t) = \int_0^\infty h(\tau)x(t - \tau)d\tau. \quad (4.23)$$

From this expression, assuming a wide-sense stationary random input, it is possible to derive relationships between some statistical moments of the input and output,

$$R_{yy}(\tau) = \int_0^\infty \int_0^\infty h(\tau_1)h(\tau_2)R_{xx}(\tau + \tau_1 - \tau_2)d\tau_1d\tau_2, \quad (4.24)$$

$$R_{xy}(\tau) = \int_0^\infty h(\tau_1)R_{xx}(\tau - \tau_1)d\tau_1. \quad (4.25)$$

Applying the Fourier Transform to Equation 4.25, it is possible to show that the frequency response $H(f)$ is given by,

$$H(f) = \frac{S_{xy}(f)}{S_{xx}(f)}. \quad (4.26)$$

The magnitude of $H(f)$ provides the amplitude response, and its phase gives the phase response of the system at frequency.

In experimental contexts, the presence of uncorrelated noise can introduce significant disturbances in the estimation of the FRF. To address this situation, various FRF estimators have been developed, each suitable for different noise scenarios. For instance, the H_1 estimator is employed when noise predominantly affects the system's output. Conversely, the H_2 estimator is used when noise is more pronounced at the input of the system. Lastly,

the H_w estimator is a valuable alternative as it takes into account the presence of noise both at the input and output of the system. The estimator is given by,

$$H_1(f) = \frac{S_{xy}(f)}{S_{xx}(f)}. \quad (4.27)$$

$$H_2(f) = \frac{S_{yy}(f)}{S_{yx}(f)}. \quad (4.28)$$

$$H_w(f) = \frac{S_{yy}(f) - S_{xx}(f) + \sqrt{(S_{xx}(f) - S_{yy}(f))^2 + 4|S_{yx}(f)|^2}}{2S_{yx}(f)}, \quad (4.29)$$

in which the PSD consider the measurement noise. The choice of the appropriate estimator depends on the specific characteristics of the experimental system and the noise distribution, ensuring precise and reliable FRF estimates under various experimental conditions. The different types of Frequency Response Functions (FRFs) play an essential role in vibration analysis. Below are the various types of FRFs commonly used in vibration analysis:

- Receptance

Mathematically defined as the ratio between the amplitudes of the output displacement $X(f)$ and the input excitation force $F(f)$, expressed by the following equation,

$$H_{re}(f) = \frac{X(f)}{F(f)}. \quad (4.30)$$

- Mobility

It defined as the ratio between the amplitudes of the output velocity $V(f)$ and the input excitation force $F(f)$, expressed by the following equation,

$$H_{mo}(f) = \frac{V(f)}{F(f)}. \quad (4.31)$$

- Accelerance

It defined as the ratio between the amplitudes of the output acceleration $A(f)$ and the input excitation force $F(f)$, expressed by the following equation,

$$H_{ac}(f) = \frac{A(f)}{F(f)}. \quad (4.32)$$

In the frequency domain, it is possible to convert all three mentioned FRFs into each other, and essentially, they share the same information. The accelerance H_{ac} can be calculated from either mobility H_{mo} or receptance H_{re} as follows,

$$H_{ac} = i\omega H_{mo} = -i\omega^2 H_{re}. \quad (4.33)$$

These FRFs can be categorized as excitation point FRFs if the excitation and response are measured at the same point, or transfer point FRFs if the excitation and response are measured at different points. The choice between these types of FRFs depends on the specific application. When mobility is taken as a reference, receptance amplifies low frequencies and attenuates high frequencies, while accelerance behaves in the opposite manner due to the mathematical operations of division and multiplication by i present in the Equation 4.33.

4.3.2 Ordinary coherence

The ordinary coherence function (γ) measures the linear relation between an input signal and an output signal in a system, in the frequency domain. It ranges from 0 to 1, indicating the degree of linear relation between the signals. The coherence function is equal to 1 in the case of $x(t)$ and $y(t)$ being linearly related; otherwise, the function equals zero if the two signals are uncorrelated. If the coherence function is greater than zero but less than one, then $x(t)$ and $y(t)$ are partially linearly related. The ordinary coherence function γ is given by the following equation,

$$\gamma(f) = \frac{|S_{xy}|^2}{S_{xx}(f)S_{yy}(f)}. \quad (4.34)$$

Coherence is useful for assessing the quality of experimental measurements and identifying the presence of nonlinearities or noise in the data. It provides information about the reliability of the relation between input and output in dynamic systems. Possible reasons for the departure from a linear relationship between the signals include the presence of noise in the measurement, $x(t)$ and $y(t)$ may not be linearly related (presence of nonlinearities), and $y(t)$ is an output affected not only by input $x(t)$ but also by other inputs.

4.4 Identification of nonlinear systems

In this section, tools for the analysis of nonlinear dynamic systems are presented, such as phase space reconstruction and the calculation of invariants, which can help to understand the underlying patterns and comprehend the nature of two-phase flow systems in pipelines.

4.4.1 Reconstruction of the phase space

The state space or phase space allows the representation of the behavior of a dynamic system in geometrical form, relating all state variables. The number of dimensions required to represent the phase space is a function of the number of degrees of freedom in

the system. In general, an experiment does not measure all system state variables, being that only one of the state variables is available. Therefore, according to [Takens \(1981\)](#) it is possible to reconstruct the original phase space from the vectors $y(t)$ extracted from a time series $x(t)$ of the system, that is,

$$y(t) = \{x(t), x(t + \tau), \dots, x(t + (d_e - 1)\tau)\}, \quad (4.35)$$

where τ is the delay time and d_e is the embedding dimension.

There are two important parameters that must be estimated correctly. First, the time delay τ , which is a measure of time that defines the basic delay. If it is too small, redundancy may occur and the points are accumulated around the bisector of the embedding space. If it is too large, they may be uncorrelated. The second important parameter is the embedding dimension d_e , which is not the dimension of the system itself, but the number of dimensions needed to correctly embed the attractor in the state space. The choice of its value is defined by Whitney's embedding theorem of $d_e \geq 2d_a + 1$, where d_a is the original dimension of the attractor.

In steady-state linear systems, the trajectories of the orbits of their phase space are repeated, while in nonlinear systems with chaotic behavior, these trajectories are never repeated, as shown in [Figure 4.1](#). However, chaotic systems have strange attractors in their phase space that attract state variables. In addition, they have an infinity of unstable periodic orbits immersed among themselves, which constitute the basic structure of chaos ([SAVI, 2006](#)).

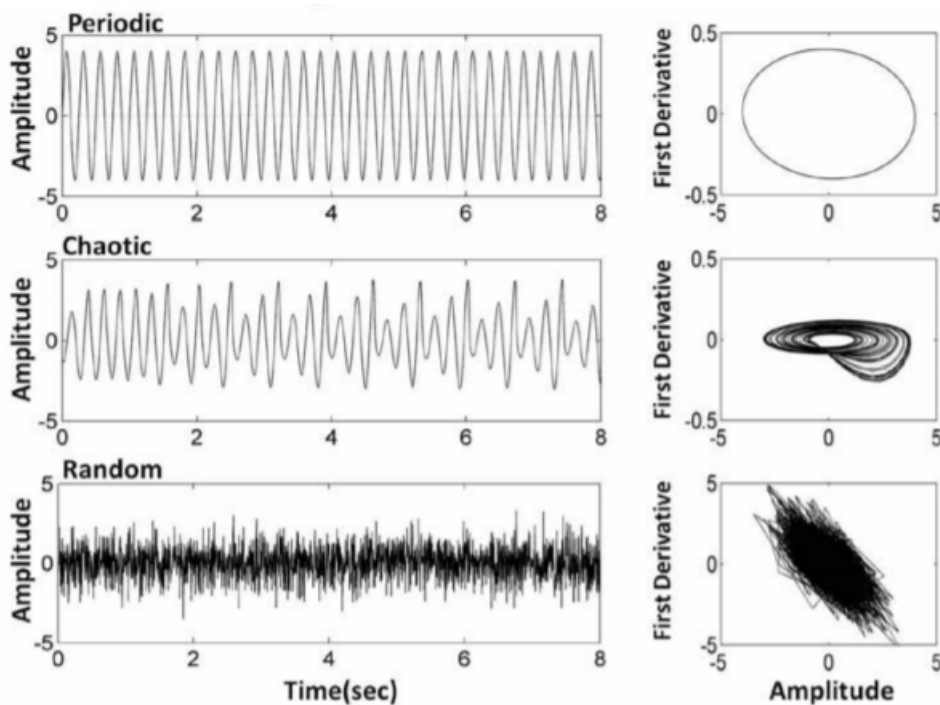


Figura 4.1 – Periodic, chaotic, and random time series (on the left) and their corresponding two-dimensional phase space plots (on the right) ([STERGIOU; YU; KYVELIDOU, 2013](#)).

From reconstructed phase space, it is possible to identify invariant characteristics on the evolution of the dynamic system and whether its formats describe qualities or properties of the system. However, the main difficulty encountered in applying this method is the need for the series to be continuous and not contaminated with noise. For experimental data, this requirement imposes a great challenge, since many types of noise are inherent to the measurement process itself.

4.4.2 Time Delay

The time delay is one the main parameter for the reconstruction of the phase space. One of the methods often used for estimating the time delay is through the Autocorrelation Function (ACF) of the time series. The ACF only measures the linear relationship between two variables; in other words, it measures the self-similarity of the signal at different delay times, in time or in space. Then, the autocorrelation function R_{yy} of time signal $y(t)$ is defined as,

$$R_{yy}(\tau) = \int_{-\infty}^{\infty} y(t)y(t + \tau)dt, \quad (4.36)$$

where $y(t + \tau)$ is the delayed version of $y(t)$.

There are different approaches to estimating τ from the ACF, for example, [King, Jones e Broomhead \(1987\)](#) considered the time of the first inflection of the ACF curve, [Zeng, Eykholt e Pielke \(1991\)](#) considered that τ is the time at which the ACF curve decays to e^1 and [Schiff e Chang \(1992\)](#) suggested using the first zero of the ACF curve to ensure linear independence of the coordinates.

Additionally, the second most used method is the Average Mutual Information (AMI), originated from information theory, it relates the amount of information that one variable contains about another. It is calculated by,

$$I(\tau) = \sum \Gamma[y(t), y(t + \tau)] \log_2 \left\{ \frac{\Gamma[y(t), y(t + \tau)]}{\Gamma[y(t)]\Gamma[y(t + \tau)]} \right\}, \quad (4.37)$$

where $\Gamma[y(t)]$ is the probability of the measure $y(t)$, $\Gamma[y(t + \tau)]$ is the probability of the measure $y(t + \tau)$, and $\Gamma[y(t), y(t + \tau)]$ is the joint probability of the measure of $y(t)$ and $y(t + \tau)$. When the measures $y(t)$ and $y(t + \tau)$ are completely independent, $I(\tau) = 0$. On the other hand, when $y(t)$ and $y(t + \tau)$ are equal, $I(\tau)$ is maximum. Therefore, the delay time is calculated from the first local minimum of curve $I(\tau)$ versus τ , because it represents the time delay in which $y(t + \tau)$ maximises the information obtained about $y(t)$ ([FRASER; SWINNEY, 1986](#)).

4.4.3 Embedding dimension

There are two main methods for determining the embedding dimension. The first was introduced by [Kennel, Brown e Abarbanel \(1992\)](#), it is called False Nearest Neighbors (FNN) procedure to obtain the optimum embedding dimension d_e . The main idea is to evaluate whether a neighbour is “true” or “false” just because of the projection of the system in a given dimension. A false neighbour is a point on the signal that only corresponds to a neighbour due to the observation of the orbits in small state space, *i.e.*, if the d_e is increased to an appropriate value, all neighbouring points of all orbits are true neighbours. It considers that the vector $y(t)$ obtained from the reconstruction, with d_e , has $y^{(r)}(t)$ close neighbours, the Euclidean distances r_{d_e} being given by,

$$r_{d_e}^2(t, r) = \sum_{k=0}^{d_e-1} \left[x(t + k\tau) - x^{(r)}(t + k\tau) \right]^2. \quad (4.38)$$

The smallest embedding dimension to represent a phase space properly occurs when the number of false neighbors is zero. So, the condition for deciding whether neighbors are false is given by,

$$\left[\frac{r_{d_e+1}^2(t, r) - r_{d_e}^2(t, r)}{r_{d_e}^2(t, r)} \right] > R_{tol}, \quad (4.39)$$

if the ratio of relative distances r_{d_e} between neighbours in the two embedding spaces is larger than a predefined value R_{tol} then the two points $y(t)$ and $y(t + \tau)$ are false neighbours. R_{tol} is the threshold to define the maximum distance growth with increasing size of dimensions and can assume values between 10 and 15 ([KENNEL; BROWN; ABARBANEL, 1992](#)). For these neighbours to be considered false, a second criterion is verified, called proximity threshold A_{tol} , and it is recommended to adopt values of approximately 2 ([KENNEL; BROWN; ABARBANEL, 1992](#)). Therefore, this method is very sensitive to noise and mainly to the value of the system time delay.

The second approach was introduced by [Broomhead e King \(1986\)](#) called the singular system analysis, which obtains the singular values and singular vectors from the snapshot matrix similar to the form of the Hankel matrix. The main feature is to obtain a noise-free unfolded attractor considering only the main singular vectors and disregarding irrelevant high-order dynamics related to lower singular values. This method also eliminates the dependency on time delay estimation considering a unit delay. For that, it is considered a matrix $n \times d_e$, where n is the length of a window moving through the data and the d_e columns of the matrix represent the number of independent variables used to define the state space. In particular, the state space is described by the matrix Y ,

$$Y = \begin{bmatrix} y_0 & y_\tau & y_{2\tau} & \cdots & y_{(d_e-1)\tau} \\ y_1 & y_{\tau+1} & y_{2\tau+1} & \cdots & y_{(d_e-1)\tau+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{n-1} & y_{\tau+n-1} & y_{2\tau+n-1} & \cdots & y_{(d_e-1)\tau+n-1} \end{bmatrix}, \quad (4.40)$$

where y is the time series measurement, n is the number of observations, d_e is the embedding dimension, and τ is the delay time.

On a mathematical basis, the matrix Y is factored into its singular values and represented by Equation 4.41, the Singular Value Decomposition (SVD). Where Λ is the diagonal matrix containing the singular values of Y in decreasing order, U and V are the matrices of the singular vectors associated with the eigenvalues in Λ . Details on the factorisation algorithm can be found in [Broomhead e King \(1986\)](#).

$$\Lambda = V^T Y U, \quad (4.41)$$

The most important characteristic is that the larger singular values in Λ are associated with dominant singular vectors, that is, those that represent the dominant features of the system dynamics, while smaller ones correspond to high-order noise-like components. [Gavish e Donoho \(2013\)](#) proposed a method to calculate a threshold value ζ for non-square matrices $m \times n$ as

$$\zeta \approx (0.56\beta^3 - 0.95\beta^2 + 1.82\beta + 1.43)y_{med}, \quad (4.42)$$

where $\beta = m/n$ and y_{med} are the median singular value of the data matrix Y , which was used by [Vieira \(2022\)](#) to define a systematic parameter for the embedding dimension.

4.4.4 Correlation dimension

The Correlation Dimension (CD) is a characteristic measure that can be used to describe the geometry of chaotic attractors in terms of self-similarities. Strange attractors with fractional dimensions are typical of chaotic systems, in other words, non-integer dimensions are assigned to geometric objects that exhibit an unusual kind of self-similarity and that show structures at all length scales. [Grassberger e Procaccia \(1983\)](#) proposed a methodology to calculate the correlation dimension given by,

$$CD(\epsilon) = \frac{1}{N(N-1)} \sum_{i=1}^N \sum_{j=i+1}^N \Theta(\epsilon - \|Y_i - Y_j\|), \quad (4.43)$$

where $\Theta(\epsilon - \|y_i - y_j\|)$ is the Heaviside function with $\Theta(x) = 1$ for $x > 0$ and $\Theta(x) = 0$ for $x < 0$, where $x = \epsilon - \|y_i - y_j\|$ and ϵ is the vector norm (i.e. radius of a sphere) centered on Y_i or Y_j , r_{min} and r_{max} were set as the minimum and maximum distances between points, respectively.

The $CD(\epsilon)$ and radius ϵ are related according to,

$$CD(\epsilon) \propto \alpha \epsilon^c, \quad (4.44)$$

where α is constant and c is the correlation dimension exponent or the slope of the logarithmic curve $CD(\epsilon)$ versus ϵ .

One can observe that two critical parameters influence the quality of this estimator. The first is N , which is delimited by the sample size. And the second is ϵ that is limited by the finite precision of the data and the inevitable lack of close neighbours on small length scales (KANTZ; SCHREIBER, 2003). To minimise these problems, Theiler (1986) proposed a new relation for the CD by fixing a minimal time separation $t_{min} = n_{min}\delta t$,

$$CD(\epsilon) = \frac{2}{(N - n_{min})(N - n_{min} - 1)} \sum_{i=1}^N \sum_{j=i+1}^N \Theta(\epsilon - \|\mathbf{s}_i - \mathbf{s}_j\|), \quad (4.45)$$

where $\|\mathbf{s}_i - \mathbf{s}_j\|$ are delay embedding vectors.

The minimal time separation t_{min} is defined by the method proposed by Provenzale et al. (1992) through the space-time separation plot. The idea is that, in the presence of temporal correlations, the probability that a given pair of points have a distance less than ϵ depends not only on the time elapsed between the two measurements. Thus, on a curve ϵ versus δt the minimum separation time corresponds to the value where the curves tend to be flat.

4.4.5 Lyapunov exponent

Lyapunov exponents measure the predictability and sensitivity of the system to changes in its initial conditions. Consider two points in the reconstructed state space, y_{n1} and y_{n2} , associated with a neighbouring and a reference trajectory, respectively, their distances are calculated as $\|y_{n1} - y_{n2}\| = \eta_0$. After time t , it is expected that the new distance will be equal to $\eta = \eta_0 e^{\lambda t}$, where λ is called the Lyapunov exponent. A positive exponent indicates that the reference and neighbouring trajectories diverge and, consequently, are associated with orbital instability and chaos. A negative exponent implies that the trajectories asymptotically tend to a common fixed point. Finally, a zero exponent means that the trajectories maintain their relative positions, thus associated with the orbital stability of deterministic systems (WOLF et al., 1985a; ROSENSTEIN; COLLINS; De Luca, 1993).

The number of Lyapunov exponents correspond to the number of dimensions of the system as they describe the expansion or contraction of the orbits for each direction. The largest Lyapunov exponent plays an important role as it provides evidence of deterministic chaos in the analysed data. [Kantz \(1994\)](#) proposed an algorithm for calculating the largest Lyapunov exponent from the reconstructed state space described by,

$$S(k) = \frac{1}{N} \sum_{i=1}^N \ln \left(\frac{1}{\|U_i\|} \sum_{j \in U_i} \|y_{i+k} - y_{j+k}^*\| \right), \quad (4.46)$$

where U_i is the number of neighbours found around point y_{i+k} and y_{j+k}^* is the reference and neighbourhood point.

A plot of the elongation factor S versus the number of k points will result in a curve with a linear rise at the beginning, followed by an almost flat region. The first part of this curve represents the exponential increase of S as more orbit points are included, while the flat region means the saturation effect of the exponential divergence given the finite size of the attractor. Thus, a linear fit to the curve produces an estimate of the greatest Lyapunov exponent.

5 Two-phase flow experimental setup

This chapter presents the experimental setup, equipment, sensors, and experimental matrix that were used in this work. Furthermore, the resistive sensor that was developed within the scope of this work will be presented, but used as a validation tool for the estimates obtained by the sensors and acceleration in the following section. The experimental setup was built at LABPETRO - "Laboratório Experimental de Petróleo Dr. Kelsen Valente", in one of the labs of Artificial Lift and Flow Assurance Research Group (ALFA) at the Center for Energy and Petroleum Studies - CEPETRO of the University of Campinas (Brazil), located at the *Universidade de Campinas* (UNICAMP).

5.1 Experimental assembly

Figure 5.1 represents the schematic drawing of the complete layout of the experimental bench. There are two different pumps for the liquid phase: a centrifugal one (Itap - 65-330/2) for high liquid flow rates and a positive displacement (Netzsch - NM090-02S) one for low flow rates. These pumps are activated individually depending on the experimental points and are controlled by a frequency inverter (WEG - CFW700). At the beginning of the linear region, a bypass for draining the flow was mounted, which will be activated during measurements of quick-closing valves, or as a pressure relief valve calibrated to 4.5 bar. The liquid is pumped from the tank to the liquid line (blue line) by one of the two pumps, and its mass flow rate is measured with a Coriolis flowmeter (MicroMotion - F300S355CCAZPZZZZ). Compressed air is supplied from the laboratory gas line (green line) at a pressure of 8 bar, a pressure regulator and needle valve control the mass flow of gas, the mass flow rate is also measured by a Coriolis flowmeter (MicroMotion - CMFS050M313N2BGPKZZ). Thus, the air is introduced in the liquid line by a simple T mixture. The experimental apparatus consists entirely of a 3"-inch pipe, with 24 meters free for the development of the flow before the measurement section, the distance between points A and B. Between points, A and C have 36.72 m. For safety reasons, the line is equipped with a by-pass that ensures that the test section pressure is always below 2 bar.

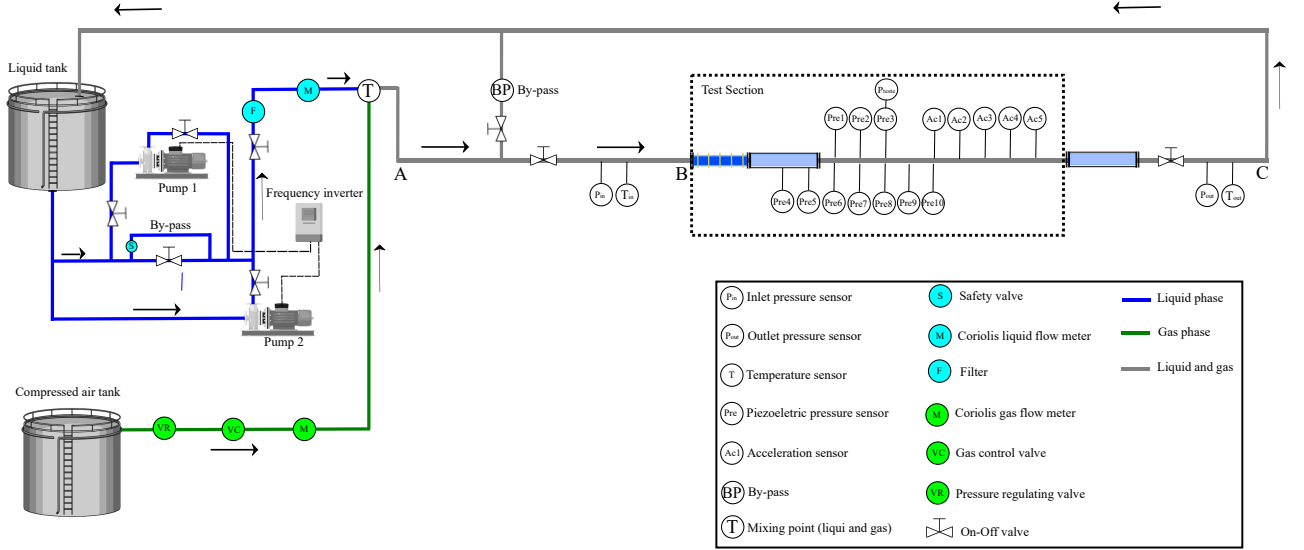


Figure 5.1 – General arrangement of the experimental apparatus, highlighting the passage of compressed air (green line), the liquid phase (blue line), and the two-phase mixture (gray line). The test section is highlighted and shown in Figure 5.1.

Tables 1 and 2 show the properties of each phase and the duct, respectively. However, note that important measurements are also taken in Figure 5.1, such as manometric pressure, measured before, after, and in the test section. Temperature is also measured at the inlet and outlet of the test section. These measurements are important to correct the density property of the gas phase, for this reason, the density measurement is not presented in Table 1. The pipe density is estimated from a pipe sample using its weight in a precision scale, then the Young’s modulus is adjusted from a modal analysis of the pipe test section. Finally, the measurement of the pressure drop is carried out by different sensors positioned in the test section, with a distance of 3 m between the measurement sockets.

Tabela 1 – Properties of the liquid and gas phases.

Property	Liquid phase	Gas phase
Density, kg/m^3	998	-
Viscosity, cP	0.001	0.000018
Superficial tension, N/m	0.0077	0.0077

Tabela 2 – Property and geometry of the steel pipe.

Property	Pipe	Geometry	Pipe
Density, kg/m^3	7337.86	Internal diameter, m	0.08050
Elasticity modulus, GPa	204.40	Thickness, m	0.00425
Roughness	0.00060	Inclination, rad	0

Figure 5.2 presents the test section with a void fraction sensor with four probes R_1 , R_2 , R_3 , and R_4 . The section of visualisation is made of a mass acrylic box, in which the place

marked with a red “x” is the sensor where the high-velocity camera was positioned. There are two models of pressure sensors with two different assemblies. The first piezoelectric pressure sensors model 1 (Piezotronics PCB 112B21) are the P_{re1} , P_{re2} , and P_{re3} sensors. The other sensors are model 2 (Piezotronics PCB 106B50), are the P_{re4} , P_{re5} , P_{re6} , P_{re7} , P_{re8} , P_{re9} and P_{re10} . Finally, five acceleration sensors of the same model (Piezotronics PCB 352C33) were used. Figure 5.3 presents a photo of the visualisation section in which it is possible to observe the resistive sensor and the high-velocity camera positioned in front of the visualisation section.

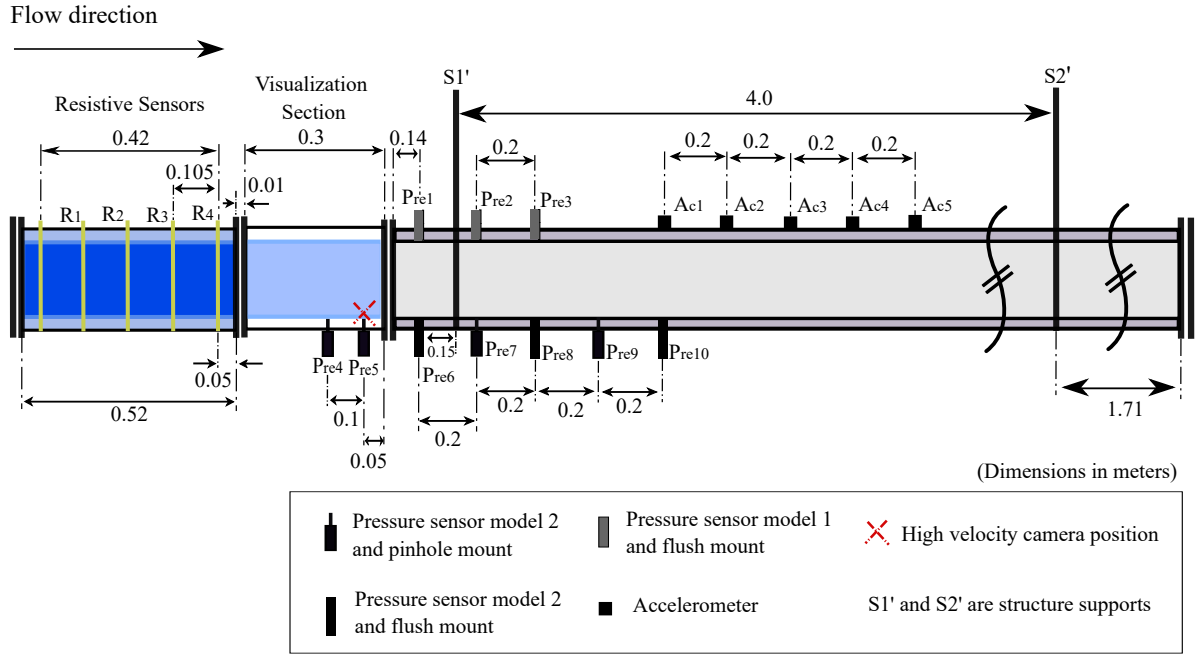


Figure 5.2 – Schematic of the test section on the experimental setup. The positions and distances of the pressure and void fraction sensors are indicated as well as the visualisation section where the high-velocity camera was positioned.

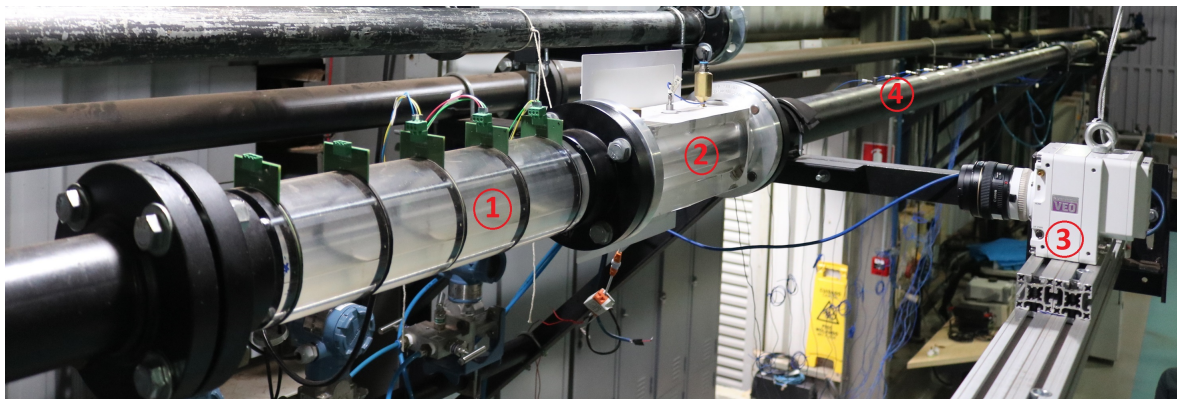


Figure 5.3 – Photo of the test section in which it is possible to observe the resistive sensor (1), visualisation section (2), high-velocity camera (3), and the steel pipe with pressure and acceleration sensors (4).

The signals from the acceleration, pressure, and void fraction sensors generated from the resistive sensors were acquired synchronously at an acquisition rate of 25.6 kHz

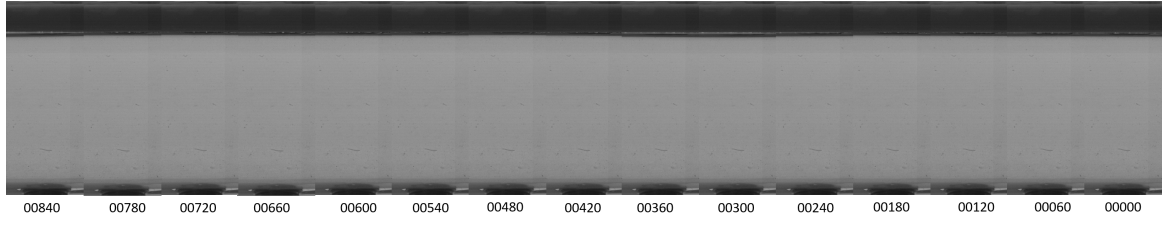
with a time of 600 seconds. The acquisition system used was from National Instruments (NI) with an interface with the Labview software. The images were acquired with a high-velocity camera (Phanton model VEO 640) at a rate of 800 fps, a resolution of 800 x 2000, and with an average recording time of 15 seconds. The sensor time series and images are synchronized in time using the trigger signal.

5.2 Test matrix

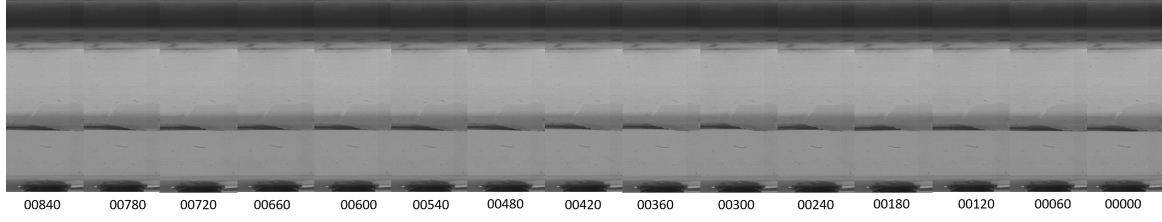
Flow patterns describe how the liquid and gas phases interact and distribute within a conduit or pipe. However, there is a difficulty in objectively defining the two-phase flow pattern and confirming it, which is related to the complexity of the interactions between the two fluids and changes in flow conditions, such as velocity, pressure, and phase proportion. For this reason, in this work, the direct visualisation method using the high-velocity camera and direct measurement using resistive sensors are used to confirm the flow patterns. In experimental tests, with the configuration and mixtures detailed previously, it was possible to observe four flow patterns. Table 3 shows the 23 experimental points acquired in the experimental test, where the liquid superficial velocities (J_{sl}), gas superficial velocities (J_{sg}), mixture velocities (J_m) and the flow pattern are detailed. Furthermore, the translational velocity (V_{TB}), slug frequency (f_{slug}) and void fraction (α_{TB}) obtained through the flow images are shown. The flow pattern was defined by observing the images from the high-velocity camera and the signals from the resistive sensors, which will be detailed in the next section. Note that at points P1 to P7 there are no parameters f_{slug} , α_{TB} and V_{TB} because they are the stratified pattern and these parameters are characteristic of intermittent patterns. Furthermore, there was an error in recording the video from point P12 and that is why it was not included in the table. Figure 5.4 shows 15 frames obtained from a high-velocity camera, in which it is possible to verify the Smooth-Stratified pattern (SS), Stratified-Wavy (SW), Slug (SL) and Dispersed Bubbles (DB).

Tabela 3 – Experimental test matrix with liquid superficial velocities (J_{sl}), gas superficial velocities (J_{sg}), mixture velocities (J_m), translational velocity (V_{TB}), slug frequency (f_{slug}), void fraction (α_{TB}) and flow pattern observed by high-velocity camera. The flow patterns are Smooth-Stratified (SS), Stratified-Wavy (SW), Slug (SL), and Dispersed Bubble (DB).

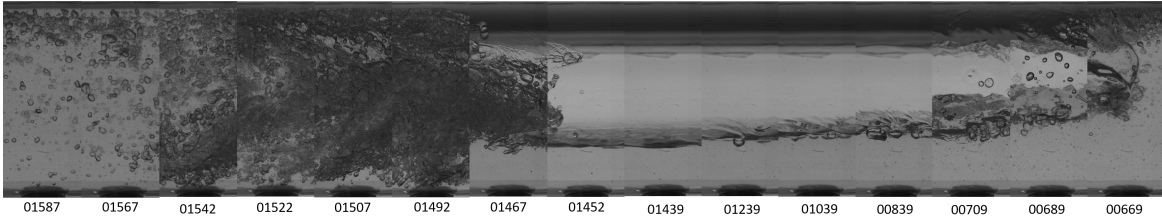
Point	J_{sl} , m/s	J_{sg} , m/s	J_m , m/s	V_{TB} , m/s	f_{slug} , Hz	α_{TB}	Flow pattern
1	0.258	0.106	0.364	-	-	-	SS
2	0.259	0.203	0.461	-	-	-	SS
3	0.258	0.481	0.738	-	-	-	SS
4	0.258	0.899	1.157	-	-	-	SW
5	0.258	1.145	1.402	-	-	-	SW
6	0.258	1.822	2.079	-	-	-	SW
7	0.259	2.036	2.295	-	-	-	SW
8	0.580	0.504	1.083	1.410	0.194	0.565	SL
9	0.579	0.977	1.556	2.090	0.388	0.636	SL
10	0.578	1.429	2.008	2.960	0.388	0.734	SL
11	1.191	0.382	1.572	2.090	0.905	0.624	SL
12	1.315	0.845	2.159	-	-	-	SL
13	1.353	1.400	2.756	3.420	0.970	0.752	SL
14	1.300	0.493	1.793	2.170	1.035	0.653	SL
15	1.242	1.001	2.243	2.750	0.841	0.692	SL
16	1.212	1.218	2.429	2.960	0.776	0.725	SL
17	2.150	0.532	2.681	3.130	2.005	0.587	SL
18	2.044	1.017	3.060	3.520	2.005	0.660	SL
19	2.017	1.182	3.199	3.750	1.746	0.656	SL
20	3.153	0.095	3.248	-	-	-	DB
21	3.042	0.421	3.462	4.020	3.298	0.548	SL
22	2.927	0.854	3.782	4.690	2.716	0.640	SL
23	3.328	0.078	3.406	-	-	-	DB



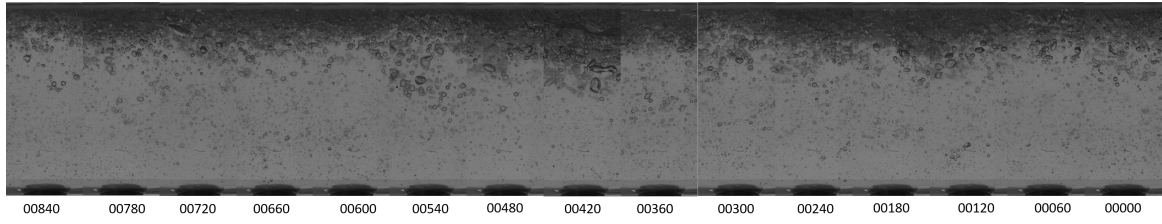
(a) P1 SS



(b) P7 SW



(c) P13 SL



(d) P20 DB

Figure 5.4 – Images obtained by filming the two-phase flow with a high-velocity camera, the pattern are Smooth-Stratified (a), Stratified-Wavy (b), Slug (c), and Dispersed Bubble (d).

The flow pattern map presented at Figure 5.5 was estimated using the model proposed by Barnea (BARNEA, 1987), in which it incorporates variables such as flow rates, fluid properties, tube diameter and tube inclination in the flow pattern transitions. The circles with black outlines represent the experimental points, whose superficial velocity values of liquid (J_{sl}) and gas (J_{sg}) and flow patterns experimentally observed by the high-velocity camera are detailed in Table 3. Note that there is an inconsistency between the patterns estimated by the map and those confirmed by the videos. The experimental points from P1 to P6 were classified as slug on the flow pattern map, but smooth-stratified and stratified-wavy patterns were experimentally verified. Also, it is possible to observe that P21 and P22 are very close to the transition to the dispersed bubble in the flow pattern map, the result is consistent with the obtained experimentally, where these points are located very close to the DB points. Therefore, the flow pattern map classified the points in the stratified pattern erroneously. This is expected in most cases close to the transition

between patterns, which depends on several closure correlations and are constrained by available experimental data, highlighting the relevance of this work. The experimental data of this article might be used to improve Barnea’s model, but it is out of the scope.

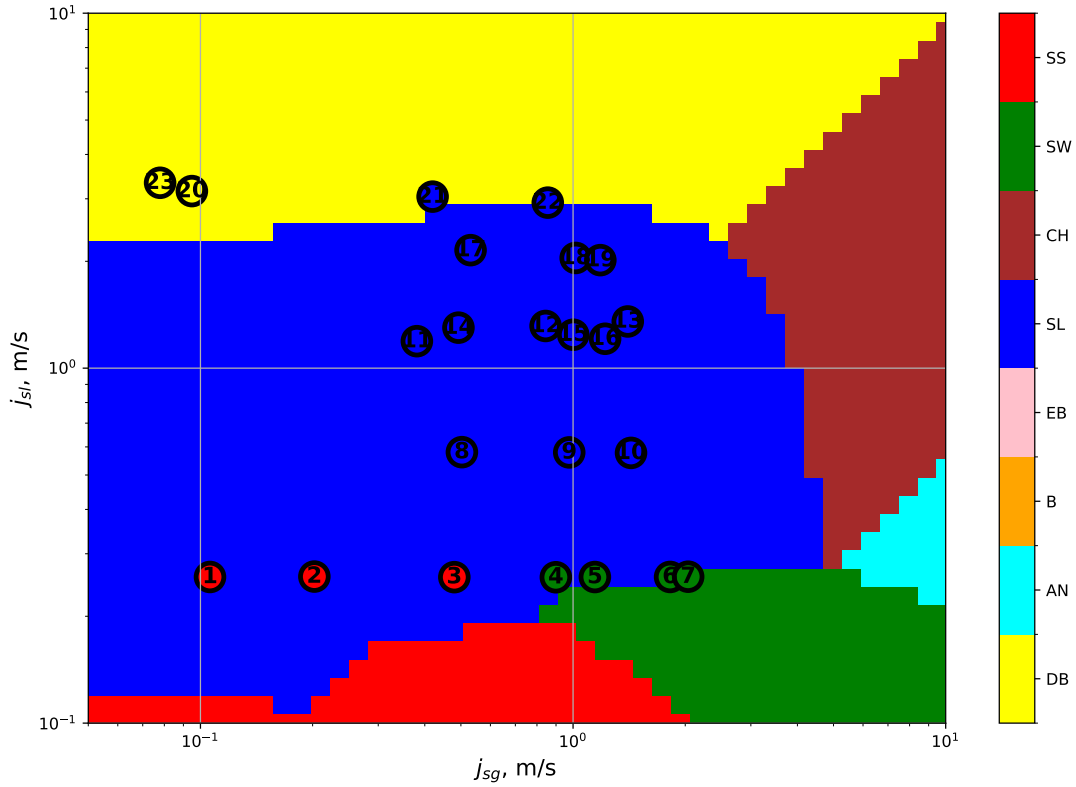


Figura 5.5 – Flow pattern map given by Barnea’s (BARNEA, 1987) model. The colors indicate the different flow patterns and the black dots indicate each point of the experimental matrix sequentially numbered according to Table 13. The flow patterns in the map are Smooth-Stratified (SS), Stratified-Wavy (SW), Churn (CH), Slug (SL), Elongated Bubble (EB), Bubbles (B), Annular (AN) and Dispersed Bubble (DB).

The liquid and gas velocities shown in this section were calculated using the measurements of average mass flow rate, density and cross-sectional area of the duct.

5.3 Estimates from resistive sensor

Resistive sensors, or conductance-based void meters, are devices used to measure the void fraction or liquid height in two-phase pipeline flows. These sensors use principles such as electrical conductivity or permittivity to make these measurements. Its operating principle consists of transmitter and receiver wires arranged perpendicularly. An oscillating electrical signal is applied to the transmitting wire and current flows from the transmitting wire to the receiving wire through a sensitive volume around the crossing point of the two

wires. Based on the current received by the receiver wires, it is possible to measure the conductivity or permittivity of the sensitive volume (RODRIGUES et al., 2020; SHI et al., 2021). However, there may be effects of residual polarisation of the electrodes that may affect sensor measurements, as well as cross-talking between subsequent sensor, resulting in systematic errors and dynamical distortions in the void fraction measurements. Because of that, a new sensor array is developed along with its dynamical characterisation to eliminate this issues, described in detail in Appendix A.

Table 4 presents some estimates taken from the void fraction signals, such as liquid piston length (L_{LP}), Taylor bubble length (L_{TB}), translational velocity (V_{TB}), slug frequency (f_{slug}) and void fraction (α_{TB}). The translation velocity was calculated using the delay time between two nearby resistive probes through the cross-correlation function. The slug frequency was estimated using the maximum peak amplitude of the auto Power Spectral Density (PSD) signal. The average void fraction was estimated using the Otsu method over the entire void fraction time series. Finally, the length of the liquid piston and the Taylor bubble were also estimated using the Otsu threshold method together with the V_{TB} information (VIEIRA et al., 2021). The resistive sensors provide direct measurements of the void fraction, making them a preferred choice due to their higher accuracy. For this reason, they are used as a reference point for comparison with the estimates provided by the acceleration methods in the next chapter.

Tabela 4 – Estimates calculated from void fraction measurements with resistive sensors: translational velocity (V_{TB}), slug frequency (f_{slug}), void fraction (α_{TB}), Taylor bubble length (L_{TB}) and liquid piston length (L_{LP}).

Point	J_{sl} , m/s	J_{sg} , m/s	V_{TB} , m/s	f_{slug} , Hz	α_{TB}	L_{TB} , m	L_{LP} , m
1	0.258	0.106	-	-	-	-	-
2	0.259	0.203	-	-	-	-	-
3	0.258	0.481	-	-	-	-	-
4	0.258	0.899	-	-	-	-	-
5	0.258	1.145	-	-	-	-	-
6	0.258	1.822	-	-	-	-	-
7	0.259	2.036	-	-	-	-	-
8	0.580	0.504	1.466	0.166	0.483	4.897	3.765
9	0.589	0.977	2.137	0.133	0.595	7.736	2.507
10	0.578	1.429	2.742	0.300	0.699	8.178	1.610
11	1.191	0.382	1.970	0.965	0.549	0.731	1.247
12	1.315	0.845	2.471	0.832	0.612	1.381	0.995
13	1.353	1.400	3.077	1.265	0.693	2.269	1.057
14	1.300	0.493	2.033	0.932	0.585	0.912	1.089
15	1.242	1.001	2.574	0.732	0.664	2.082	1.130
16	1.212	1.218	2.800	0.566	0.703	2.629	1.193
17	2.150	0.532	2.933	2.063	0.591	0.541	0.868
18	2.044	1.017	3.410	1.697	0.644	0.974	0.907
19	2.017	1.182	3.500	1.531	0.663	1.131	0.916
20	3.153	0.095	-	-	-	-	-
21	3.042	0.421	3.818	2.762	0.537	0.477	0.732
22	2.927	0.854	4.200	2.679	0.621	0.643	0.953
23	3.328	0.078	-	-	-	-	-

In Table 4, it is evident that the experimental points P1 and P2 display significantly low void fraction values, residing at the lower operational limit of these resistive sensors in void fraction measurement. Consequently, the measurements taken at these points lack precision and reliability, as they operate at the sensor's operational limits.

6 Results and Discussions

In this chapter, the results derived from the analysis of pressure and acceleration signals collected from the experimental setup described in Chapter 5, will be presented and discussed. The primary aim of this analysis is to investigate the fluid-structure coupling involving pressure and structural waves and its relation with the void fraction dynamics. Through this phenomenon, it is possible to identify flow patterns, estimate the frequency of slug patterns, determine the velocity of Taylor bubbles, and track the trajectory of Taylor bubbles within slug patterns from indirect measurements, given by pressure and/or acceleration, which are non-invasive and non-intrusive, respectively. Additionally, a nonlinear system identification analysis is introduced, employing acceleration and pressure signals to characterise the intrinsic dynamics of each flow pattern. Finally, a methodology for flow pattern identification is proposed, using chaotic system metrics on the pressure signals.

6.1 Time-frequency analysis

In this section, an introductory analyses of the signals in both the time and frequency domains will be discussed, with a particular focus on acceleration signals. Additionally, the signals in the time domain, synchronised using the time scale of the translational velocity of Taylor bubble (V_{TB}), estimated from the void fraction signals, will be addressed.

6.1.1 Frequency domain analysis

Figures 6.1 presents the spectrogram and time series of acceleration signal for experimental point P13 in the slug pattern, with the sampling rate of 25600 Hz. The spectrograms were obtained using a Short-Time Fourier Transform approach (STFT) and are calculated with 90 segments, 50% overlapping, and Hann windowing. From the analysis of the spectrogram, the non-stationary character is noted in the time of the acceleration signals, due to the alternating energy peaks around a central frequency. This behaviour is characteristic of the intermittent flow pattern, due to the alternating flow of two structures: liquid piston and Taylor bubble. In the test section, comprising a 6-meter-long circular duct, the analysis of low-frequency behaviour revealed the predominance of flexural modes

(MATOS et al., 2022a). These flexural modes were identified through structural assessment and manifested at specific frequencies of 12 Hz, 20 Hz, 60 Hz, and 120 Hz. In contrast, at high frequencies, the cut-on modes dominate the structure, which will be further analyzed in the following sections. In this context, the frequencies associated with these modes were measured at 1384 Hz, 3993 Hz, and 7637 Hz. Therefore, it is observed in the spectrogram that transient components in time exhibit constancy in frequency. This constancy becomes evident in the vicinity of the natural frequencies and cut-on frequencies of the structure.

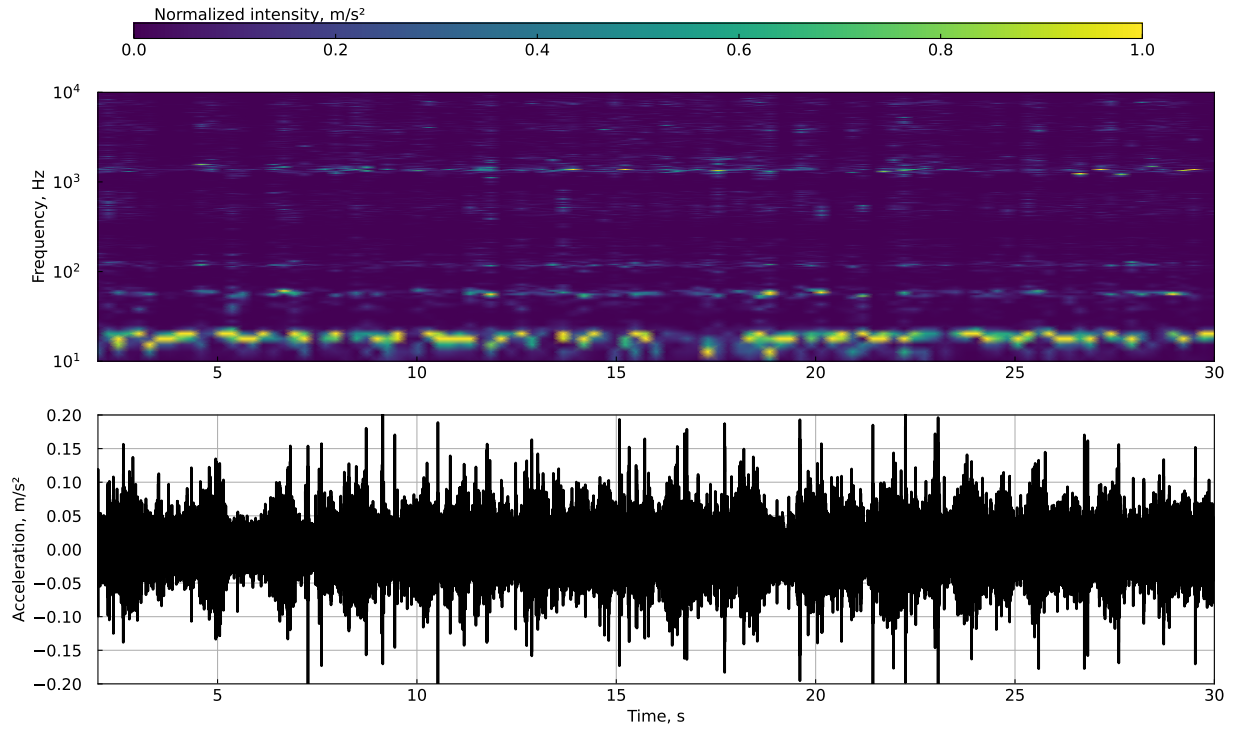


Figure 6.1 – Spectrogram and time series of the acceleration signal for experimental point P13 in the slug pattern.

In a pattern of dispersed bubbles, the moving gas bubbles, collisions, and interactions with the liquid introduce fluctuations in the fluid properties. This can lead to variable frequencies in the spectrogram, reflecting the different vibration natural frequencies induced by the moving bubbles. Figures 6.2 presents a spectrogram and acceleration time series for the dispersed bubbles pattern, calculated with the same method and parameters as Figures 6.1. Note that, similar to what is observed in the slug pattern, peaks of energy near the structural frequencies are also observed.

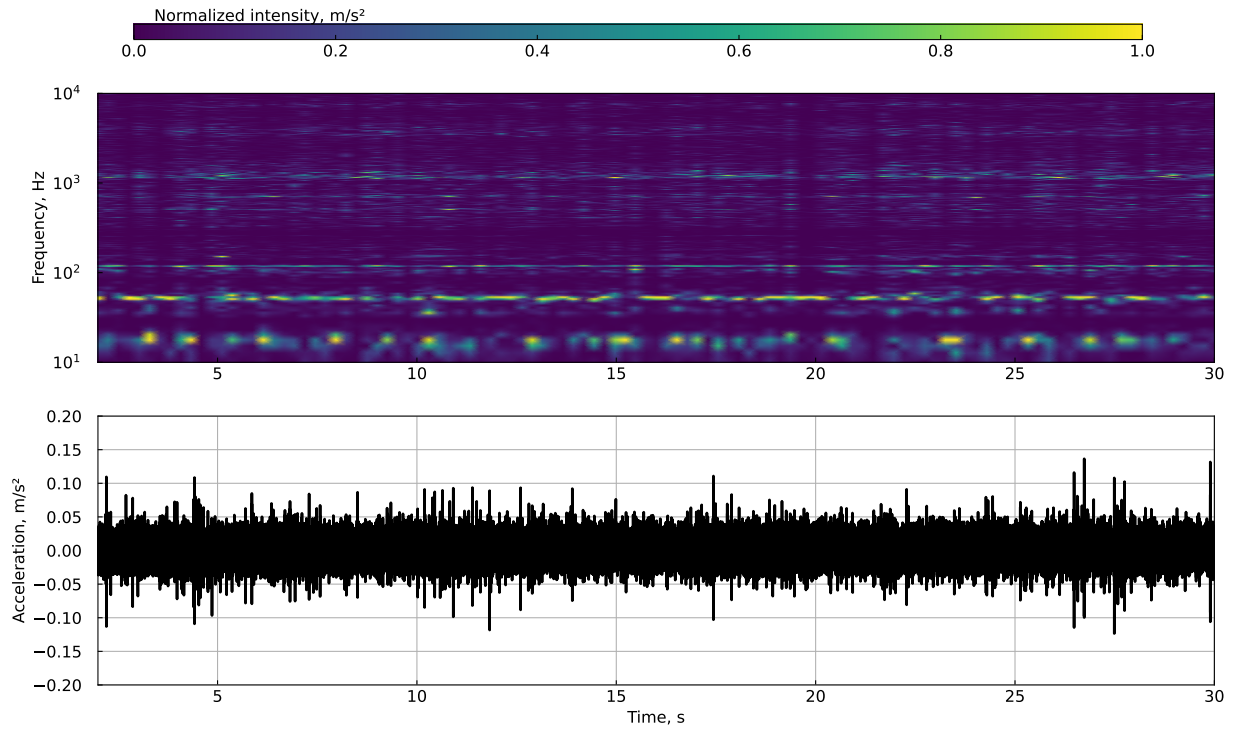


Figure 6.2 – Spectrogram and time series of the acceleration signal for experimental point P20 in the dispersed bubbles pattern.

Figure 6.3 presents a spectrogram and acceleration time series for the stratified pattern, calculated with the same method and parameters as Figures 6.1 and 6.2. Note that, unlike the previous analyses, for this flow pattern, there are no oscillatory components over time due to the inherent characteristics of this flow pattern. In a stratified flow pattern, the liquid and gas phases flow in distinct layers with clear interfaces between them. This results in a more stable dynamic behaviour with fewer oscillations compared to intermittent patterns or other more complex flow patterns. Furthermore, the presence of a consistent energy trace at 120 Hz may be attributed to other factors, such as the influence of pumps or external sources of noise.

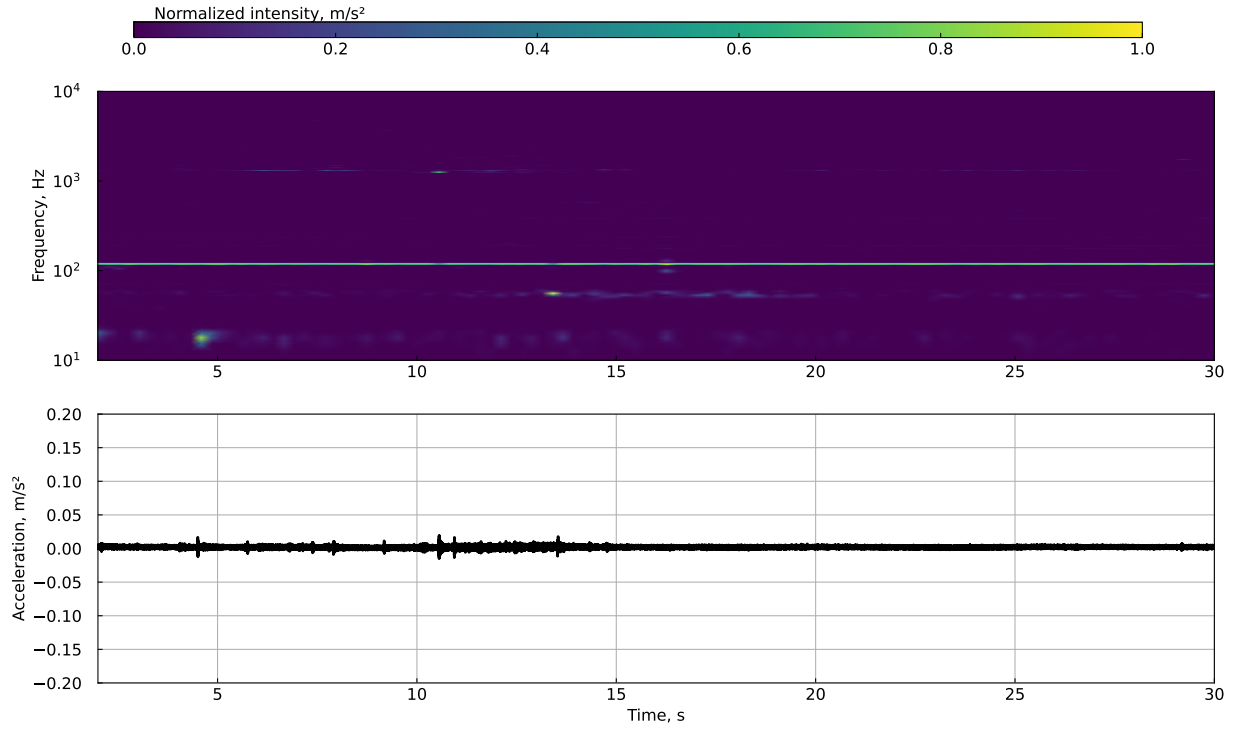


Figure 6.3 – Spectrogram and time series of the acceleration signal for experimental point P2 in the stratified pattern.

Both in the dispersed bubble patterns and slug patterns, transients are observed at the structural frequencies of the duct. A common factor among these patterns is the presence of dispersed gas bubbles in the disperse phase. The coupling between structural waves and those generated by the dispersed bubbles can lead to energy transfer between the fluid and structural domains, and this phenomenon will be investigated in the following sections.

6.1.2 Time domain analysis

The pressure and acceleration signals, P_{re10} and A_{c1} , originate from the same measurement point in the test section and are thus synchronised in the time domain, as shown in Figure 5.2. By using two resistive sensors, it is possible to calculate the average velocity of the Taylor bubble, as demonstrated in Table 4. This enables the determination of the average delay and the synchronisation of the void fraction signals with the pressure and acceleration sensors. Figures 6.4 (a) and (b) depict the synchronised acceleration and pressure signals with the void fraction signal. It is evident the ability to identify the passage of the Taylor bubble and the liquid slug through the high and low void fraction states, respectively. However, this same assertion cannot be extended when using pressure and acceleration signals. Furthermore, Figure 6.4 (c) shows the time series of the gradient and pressure ΔP obtained from the subtraction of the signals from sensors P_{re1} and P_{re6} , arranged above and below the same cross-section of the pipe, as shown in Figure 5.2, also synchronised with the void fraction accordingly. It is possible to notice that there is

a fluctuation in the pressure gradient across the pipe cross-section following the passage of the unit cell. In regions with a higher void fraction, such as the Taylor bubble, the gradient tends to zero, whereas in regions with a lower void fraction, like the liquid slug, the pressure gradient experiences a considerable increase. This pressure fluctuation is one of the mechanisms responsible for modulating the vibration signal as the unit cell passes through.

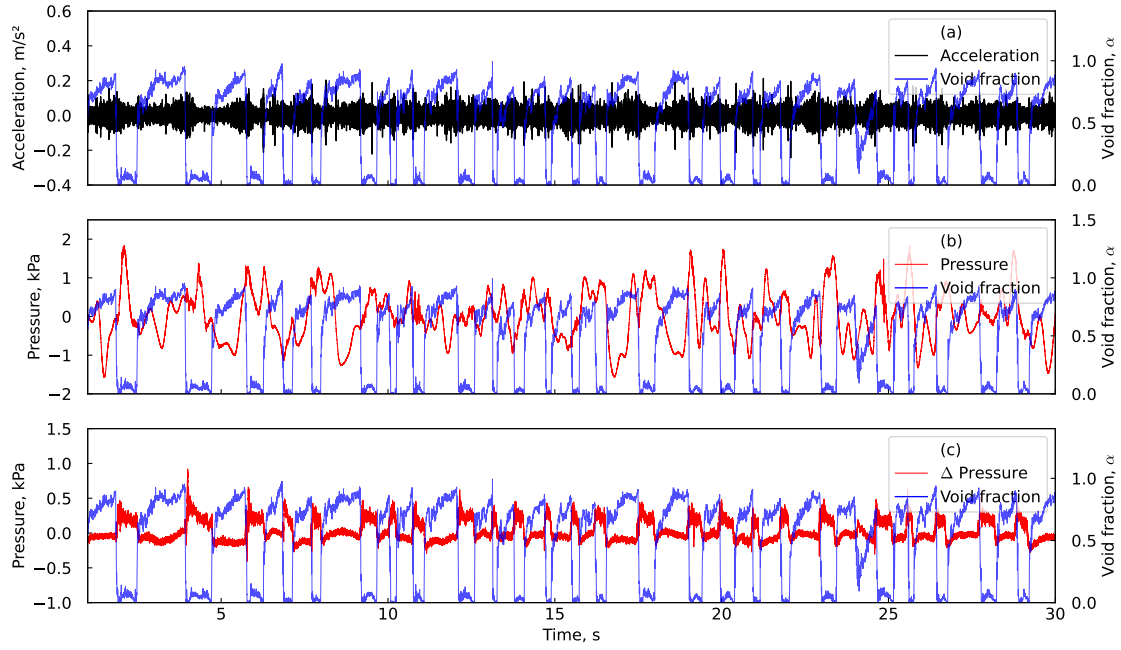


Figure 6.4 – Synchronized time series of void fraction and (a) acceleration, (b) pressure, and (c) pressure gradient for experimental point P13 in the slug pattern.

Figures 6.5 and 6.6 present the time series of void fraction and the signals of (a) acceleration, (b) pressure, and (c) pressure gradient, corresponding to patterns of dispersed bubbles and stratified flow, respectively. A fundamental observation is that in both cases, the void fraction remains close to zero, indicating the predominance of liquid within the pipe. This reflects the unique characteristics of these flow patterns. In the case of the dispersed bubble pattern, it is important to note that the void fraction of the bubbles within the liquid is extremely low, operating at the lower detection limit of the void fraction sensor. On the other hand, for the stratified pattern, the results are consistent with the data presented in Table 4. The pressure time series suggests that the stratified pattern does not exhibit significant oscillations in the temporal data, in agreement with its more stable nature, where the liquid and gas phases flow in distinct layers and maintain clear interfaces between them. On the other hand, the dispersed bubble pattern displays intense variations due to collisions and interactions between the gas bubbles and the liquid, resulting in significant and very rapid fluctuations in the signals.

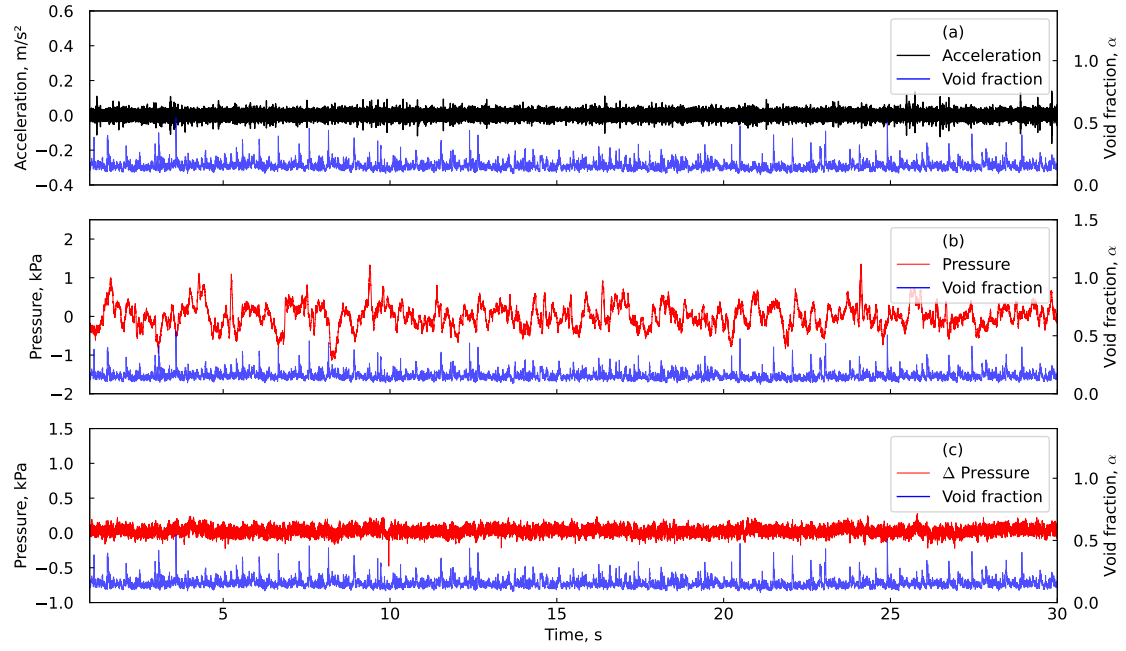


Figure 6.5 – Void fraction time series and (a) acceleration, (b) pressure, and (c) pressure gradient for experimental point P20 in the dispersed bubbles pattern.

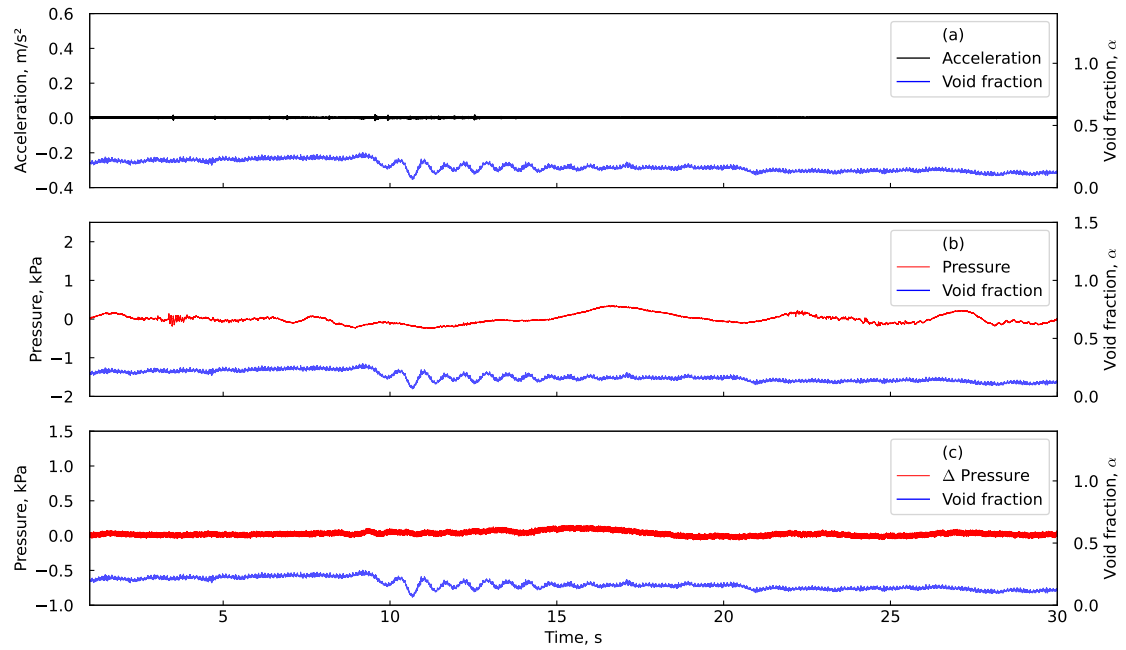


Figure 6.6 – Void fraction time series and (a) acceleration, (b) pressure, and (c) pressure gradient for experimental point P2 in the stratified pattern.

6.2 Fluid-structure coupling

The void fraction time samples are classified in Taylor bubble or liquid piston following Otsu’s method, which is an unsupervised and non-parametric approach, using R_4 conductance-based void fraction meter signal (VIEIRA et al., 2023). The result is a

binarized time series, which is subsequently synchronised with Ac_1 , P_{re10} , and the high-velocity camera, based on the time delay estimated from the Taylor bubble velocity. Thus, it is possible to determine the vibration and pressure instants in which either the liquid slug or the Taylor bubble is at the sensor position. Notice that each individual unit cell can present slight deviations from this average velocity due to hydrodynamics of the flow, as an increase in slip by the amount of gas and liquid, for example. However, the average is used as the best estimate of such velocity.

First, the Bubble Size Distribution (BSD) is estimated from the images recorded by a high-velocity camera using the Segment Anything Model (SAM), as detailed in Section B. A time series for the average diameter and the number of bubbles is used along with an estimated probability distribution to investigate the main frequencies of the flow. Subsequently, the pressure and acceleration auto and cross Power Spectral Density (PSD) are estimated for the slug and the Taylor bubble regions, individually, and for the unit cell. Then an input/output relation is investigated, highlighting the fluid-structure coupling at the cut-on frequencies. Subsequently, the vibration response is investigated at the cut-on frequency.

In this section, the experimental points P12, P14, and P21 are not shown because an error occurred in the triggering signal from the high-velocity camera, which makes it impossible to synchronise the video with the sensor's time series.

6.2.1 Bubble size distribution

Figure 6.7 and 6.8 (a) show a sequence of 13 frames, irregularly spaced in a unit cell, captured from the high-velocity camera for points 13 and 17 of the test matrix, respectively. Both patterns are slugs flowing from left to right and the red squares are the bounding boxes of the dispersed bubbles identified by the image processing algorithm. It also shows the time series of the number of bubbles N_b (b) and the average diameter d_{avg} (c) per frame. The dashed red line in the series corresponds to the position of the image in the series, sequentially. Note that the number of bubbles is large at the beginning of the Taylor bubble and decays exponentially in the stratified region. In the mixing region, there is an increase in the number and diameter of the bubbles. It is also observed that the average bubble diameter tends to be most significant in the region in the slug, in the mixing region, and at the beginning of the Taylor bubble, represented by the two largest peaks in the d_{avg} series.

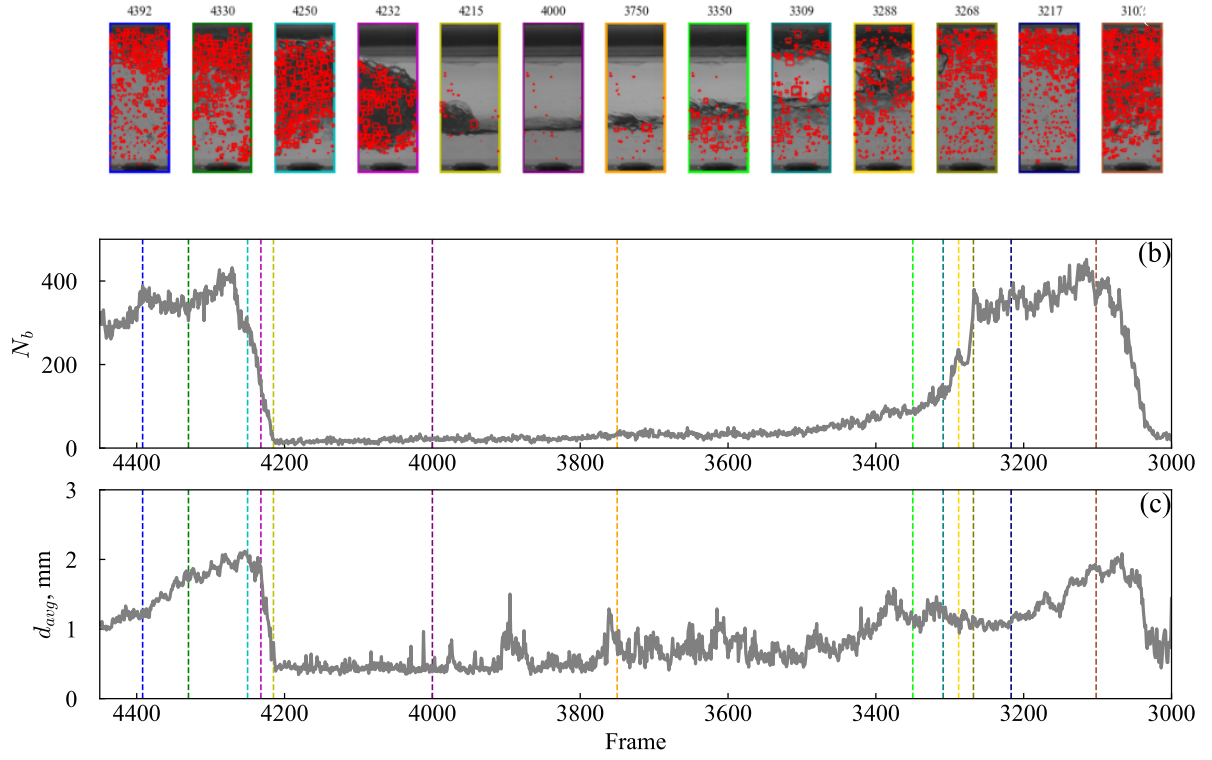


Figure 6.7 – High-velocity camera images of experimental point P13 (a), the red square is the bubbles detected by the developed algorithm. Also shown are the series of bubble numbers N_b (b) and average bubble diameter d_{avg} (c) per frame. Each red dashed line in (a) and (b) indicates the position of each frame present in (a) sequentially ordered.

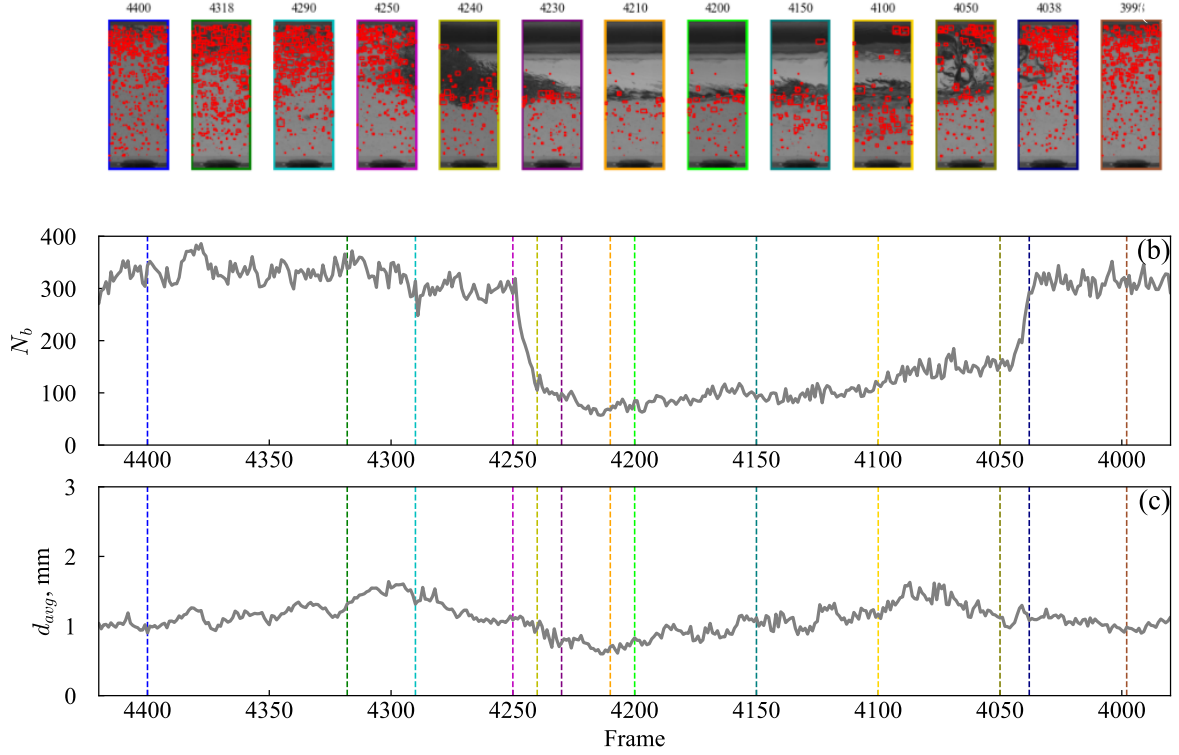


Figure 6.8 – High-velocity camera images of experimental point P17 (a), the red square is the bubbles detected by the developed algorithm. Also shown are the series of bubble numbers N_b (b) and average bubble diameter d_{avg} (c) per frame. Each red dashed line in (a) and (b) indicates the position of each frame present in (a) sequentially ordered.

Figures 6.9 and 6.10 show the void fraction series from R_4 sensor, blue line, synchronised with N_b series, grey line, and d_{avg} series, black line. Synchronisation was performed by advancing the void fraction signal from sensor R_4 to the Region of Interest (ROI) in the visualisation section. The corresponding time delay was given by Taylor bubble velocity V_{TB} estimated from the two void fraction sensors closest to the ROI, R_3 and R_4 (VIEIRA et al., 2023). It is possible to observe the evolution in time of the passage of the unit cell, when the void fraction drops the number of bubbles increases, representing the liquid piston with the small bubbles. And when the void fraction increase the number of bubble decrease representing the stratified region of the Taylor bubble. Notice that the passage frequency at experimental point 17 is greater than at experimental point 13, thus there are several more unit cells in the same span of time. Furthermore, at experimental point 13 the Taylor bubble is twice as large as the liquid piston, at experimental point 17 the slug is slightly larger than the Taylor bubble, in agreement with Table 3.

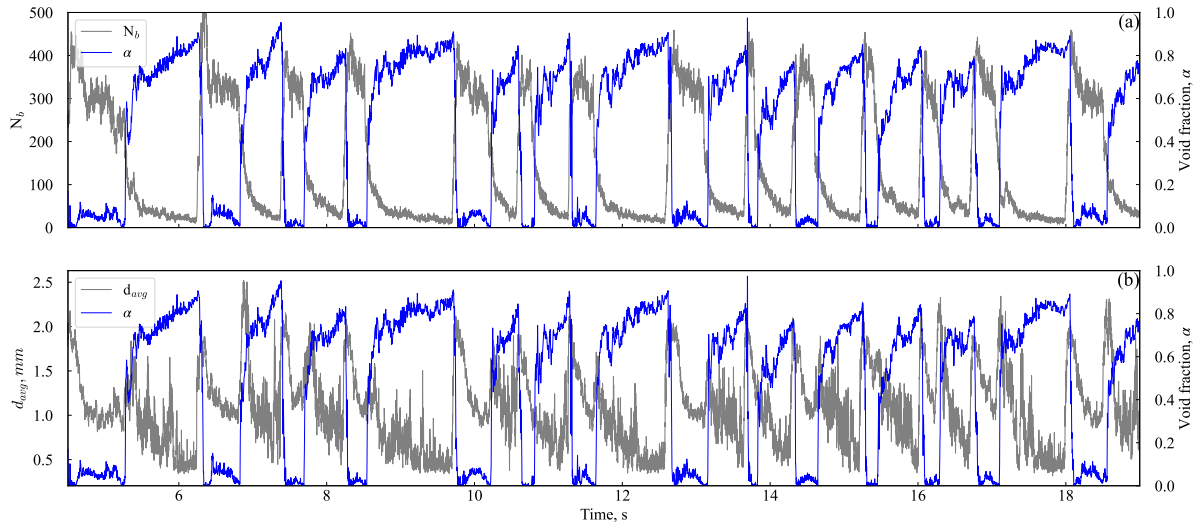


Figure 6.9 – Experimental point 13, the first graph (a) shows the void fraction time series, blue line, and the number of bubble series N_b , gray line. The second graph (b) shows the void fraction time series, blue line, and the average bubble diameter d_{avg} , gray line.

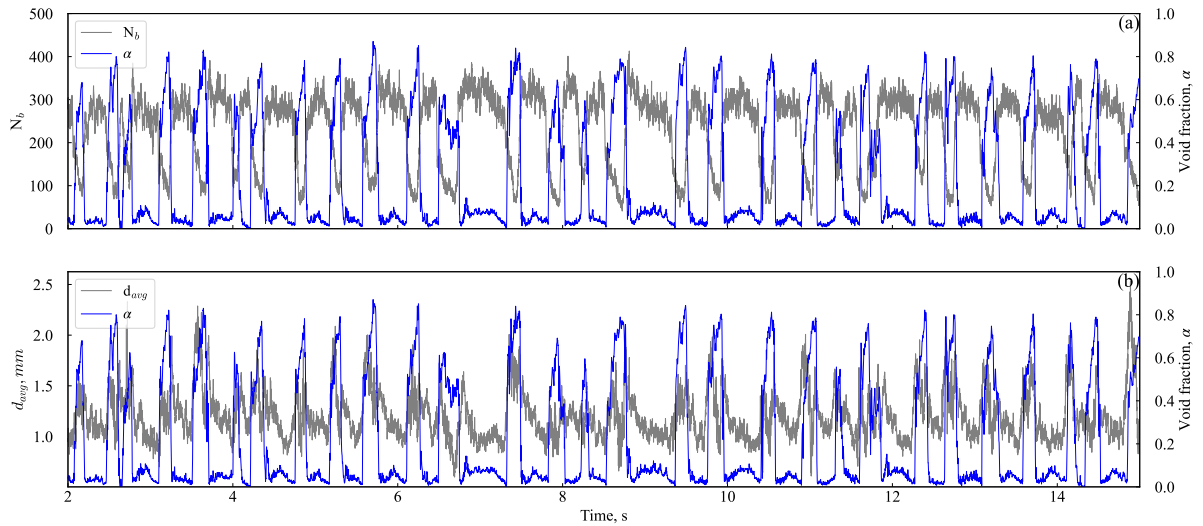


Figure 6.10 – Experimental point 17, the first graph (a) shows the void fraction time series, blue line, and the number of bubble series N_b , gray line. The second graph (b) shows the void fraction time series, blue line, and the average bubble diameter d_{avg} , gray line.

To estimate the probability distribution, the univariate Kernel Density Estimation (KDE) technique was employed using the value of each diameter and each frame identified in the image processing. In simple terms, the KDE smooths the data to generate an estimate of the Probability Density Function (PDF) that describes how the data values are distributed. Figure 6.11 presents the Bubble Size Distributions (BSD) at the unity cell (Cell), the Taylor Bubble (TB) only, and the liquid piston (LP) only, for all experimental points in the slug pattern. The synchronised and binarized void fraction time series was used to split the bubble distribution in each region of each unit cell individually. In

addition, theoretical diameters were calculated, where the upper and lower limits are the Hinze diameter (d_H) and the Brodkey diameter (d_{BK}), respectively. Within these limits, there is a region with a high rate of coalescence and collision and the bubbles are deformable. Moreover, the critical diameter (d_C) indicates the diameter from which the bubbles tend to lift toward the upper wall of the duct. It is important to highlight that each of these theoretical diameters are very close to values at where the BSD curve changes its inclination, indicating a qualitative change on the behaviour of the bubbles. The Hinze diameter is greater than the critical and Brodkey diameters for most points, except for point 22, which is close to the transition to the dispersed bubble regime, as shown in Figure 5.5. It is observed that it satisfies only one of the necessary conditions to be classified as dispersed bubbles, namely, $d_H \leq d_{BK}$ or $d_H \leq d_C$, as presented in Section 2.6.3. This suggests a special situation at this specific point, where the system's properties may be subject to significant influences from the flow pattern transition. Exploring these behaviour in details is outside the scope of this work.

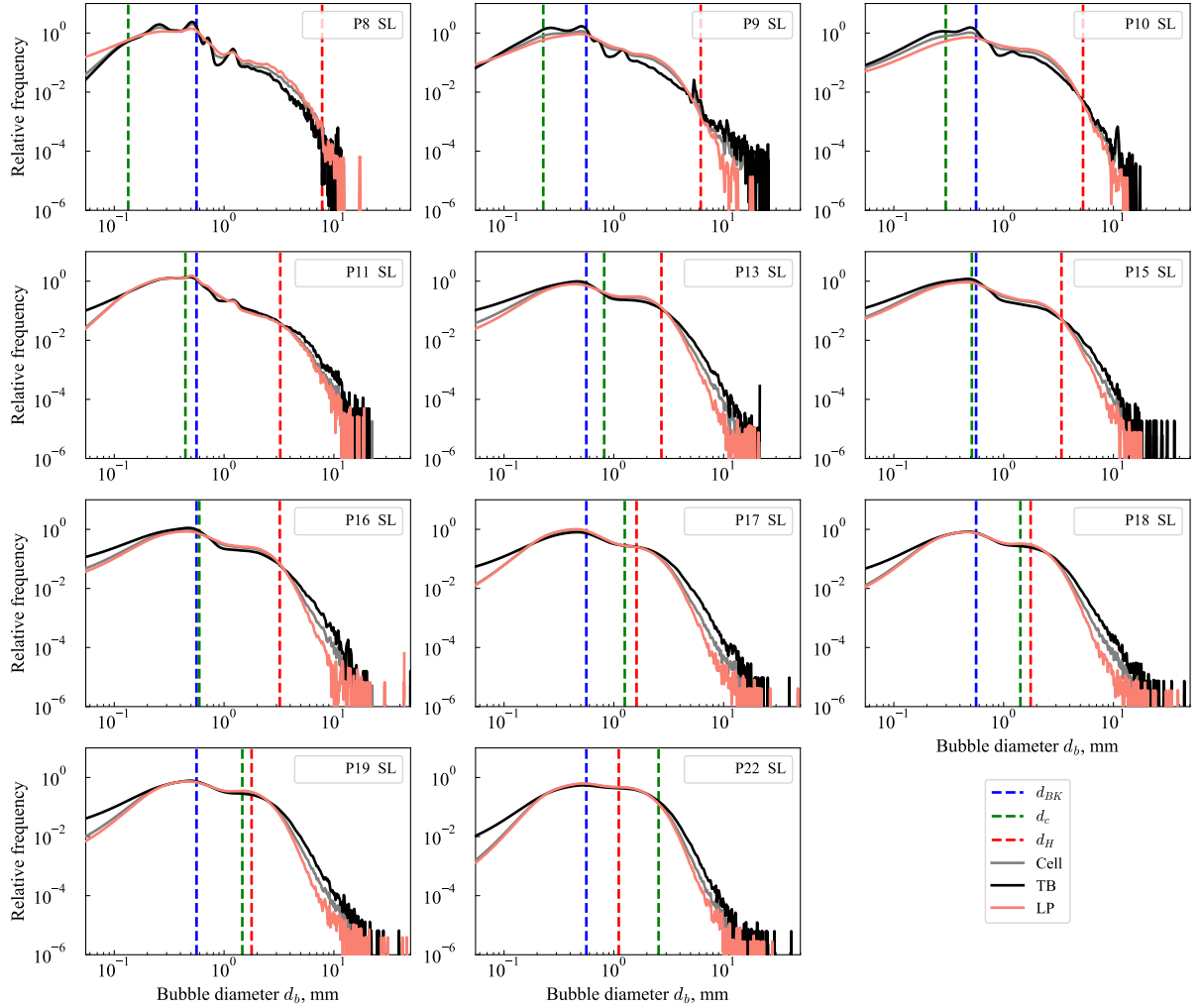


Figure 6.11 – Bubble Size Distribution (BSD) for the unit cell (Cell), the liquid piston (LP), and for the Taylor bubble (TB), for all experimental points in the slug pattern. The theoretical diameters of Hinze (d_H), Broadkey (d_{BK}) and critical (d_C) are indicated by the vertical dashed line.

In order to investigate the dynamics of vibration resonance in gas bubbles, the Rayleigh-Plesset bubble oscillator model was employed as the foundational framework. This model characterises the bubble's dynamics as that of a classical particle, establishing a correlation between liquid properties such as density and surface tension and vibration resonance. Additionally, an attempt is made to incorporate information derived from the flow itself by using the estimated diameters of bubbles resulting from image processing as input for determining the Rayleigh-Plesset resonance frequency. Figure 6.12 presents the corresponding distribution of natural frequencies calculated from the bubble diameters using the linearised Rayleigh-Plesset frequency, it calculated by Equation 2.45. It also shows f_c , f_H and f_{BK} , which are corresponding Rayleigh-Plesset frequencies for d_c , d_H and d_{BK} . Notice that the frequency distribution yields a very broad frequency band. This encompasses the first the cut-on frequency of circumferential wave modes, as presented in more detail in Section 6.2.2.

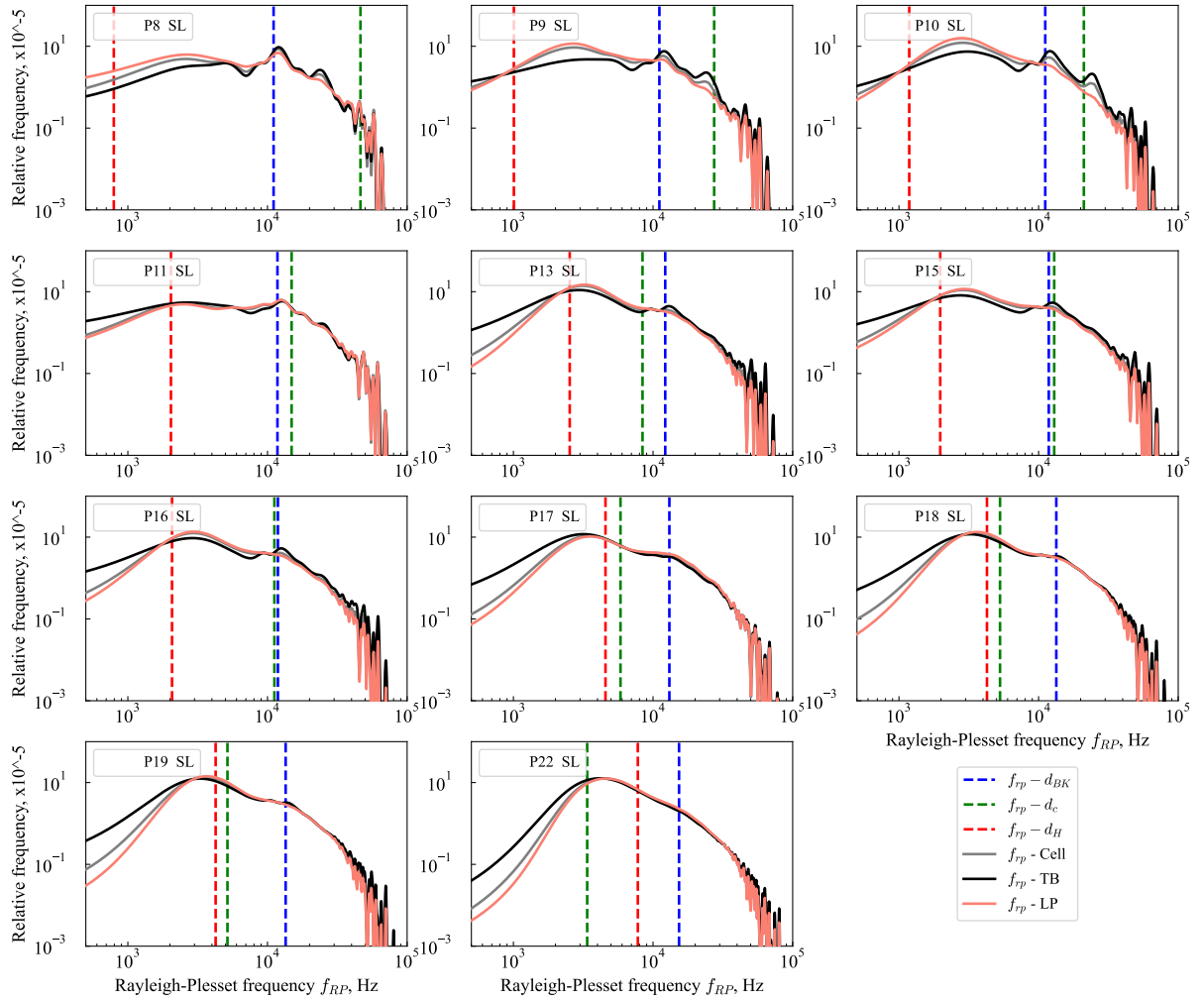


Figure 6.12 – Rayleigh-Plesset frequency distribution for the unit cell (Cell), liquid piston (LP), and for the Taylor bubble (TB), for all experimental points in the slug pattern. The Rayleigh-Plesset frequencies were calculated with the analytical diameters for the Brodkey diameter (d_{BK}), Hinze diameter (d_H) and critical diameter (d_C).

6.2.2 Fluid-structural response

In this section, the structural response is experimentally investigated in terms of pressure excitation and the acceleration response of the pipe. The accelerometer Ac_1 and the pressure sensor P_{re10} are used because they are placed at the same position on the pipe, as indicated in Figure 5.2. The cut-on frequencies of the circumferential wave modes of the corresponding empty pipe, given by Equation 3.18, for the modes $n = 2$, $n = 3$ and $n = 4$ are summarised in Table 5. In addition, the ring frequency of the empty pipe, Equation 3.17, is given at about 30 kHz.

Tabela 5 – Cut-on frequencies for the circumferential wave modes of the empty pipe.

n	2	3	4
f_n , Hz	1384	3993	7637

Figures 6.13 and 6.14 present the Power Spectral Density (PSD) estimated from the pressure and acceleration measurements, respectively, for the liquid piston (LP) only, the Taylor bubble (TB) only, and the full unity cell (Cell). They are calculated using the segment and average Welch’s periodogram. They are separated in each region of the unity cell by the synchronised binarized void fraction time series, classified in Taylor bubble or liquid piston. Each unity cell, Taylor bubble and liquid piston segment of the corresponding periodogram, is then zero padded to next power of two of the maximum number of points and detrended, with no overlapping. A Hann window with the corresponding correction for amplitude is used. The same approach is used for estimating the corresponding cross-PSDs. Notice that for both cases, the pressure and acceleration PSD of the unity cell is between the Taylor bubble and the liquid piston. Moreover, this choice of segment length yields a frequency discretizaion of the PSD of the same order of the slug frequency f_{slug} . This has the drawback of smoothing out this frequency band but significantly improves the estimated in the higher frequencies, which is the focus of this investigation.

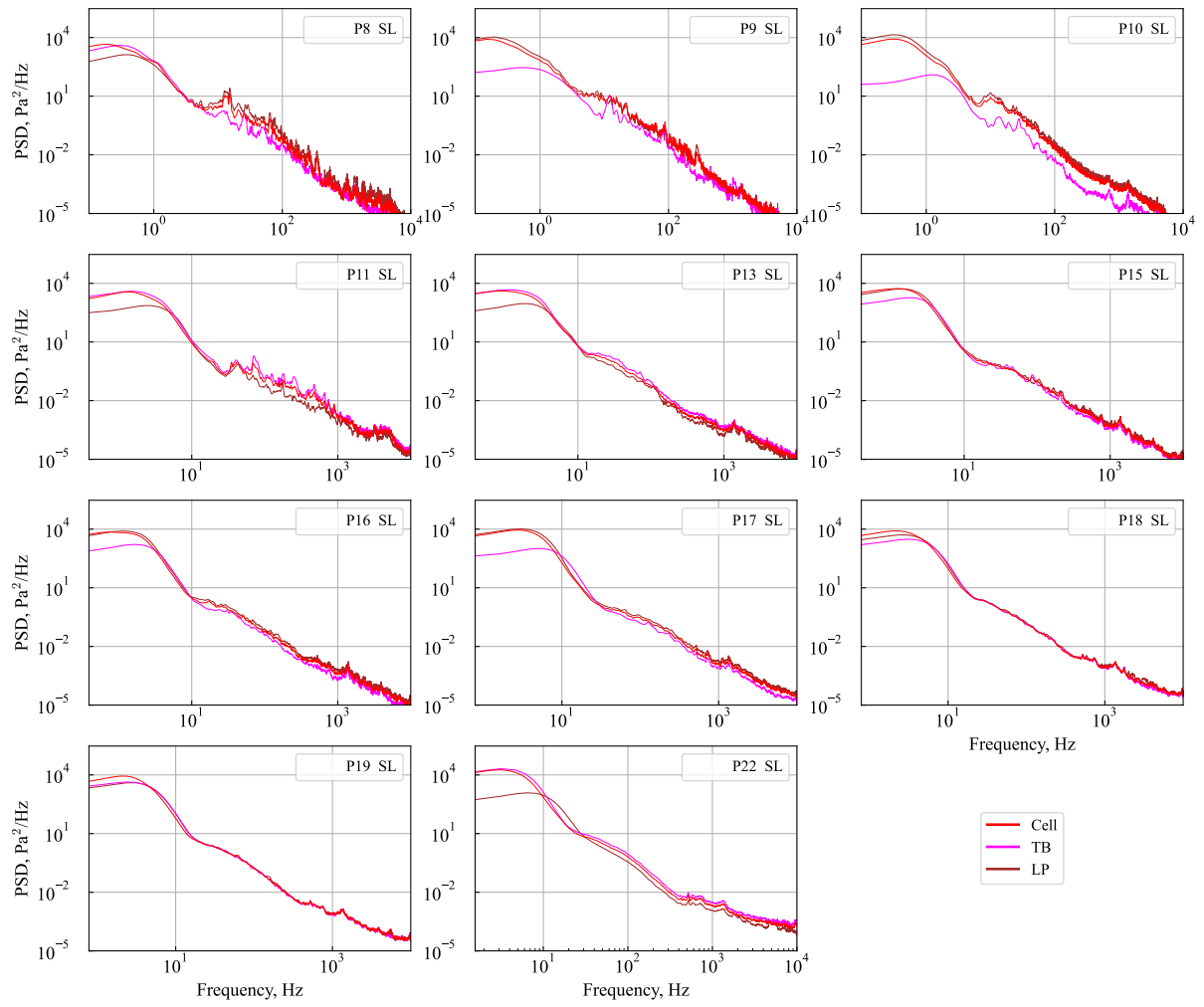


Figure 6.13 – Power Spectral Density (PSD) of the pressure measurement at the liquid piston (LP), Taylor bubble (TB) and the unit cell (Cell) for all experimental points in the slug pattern.

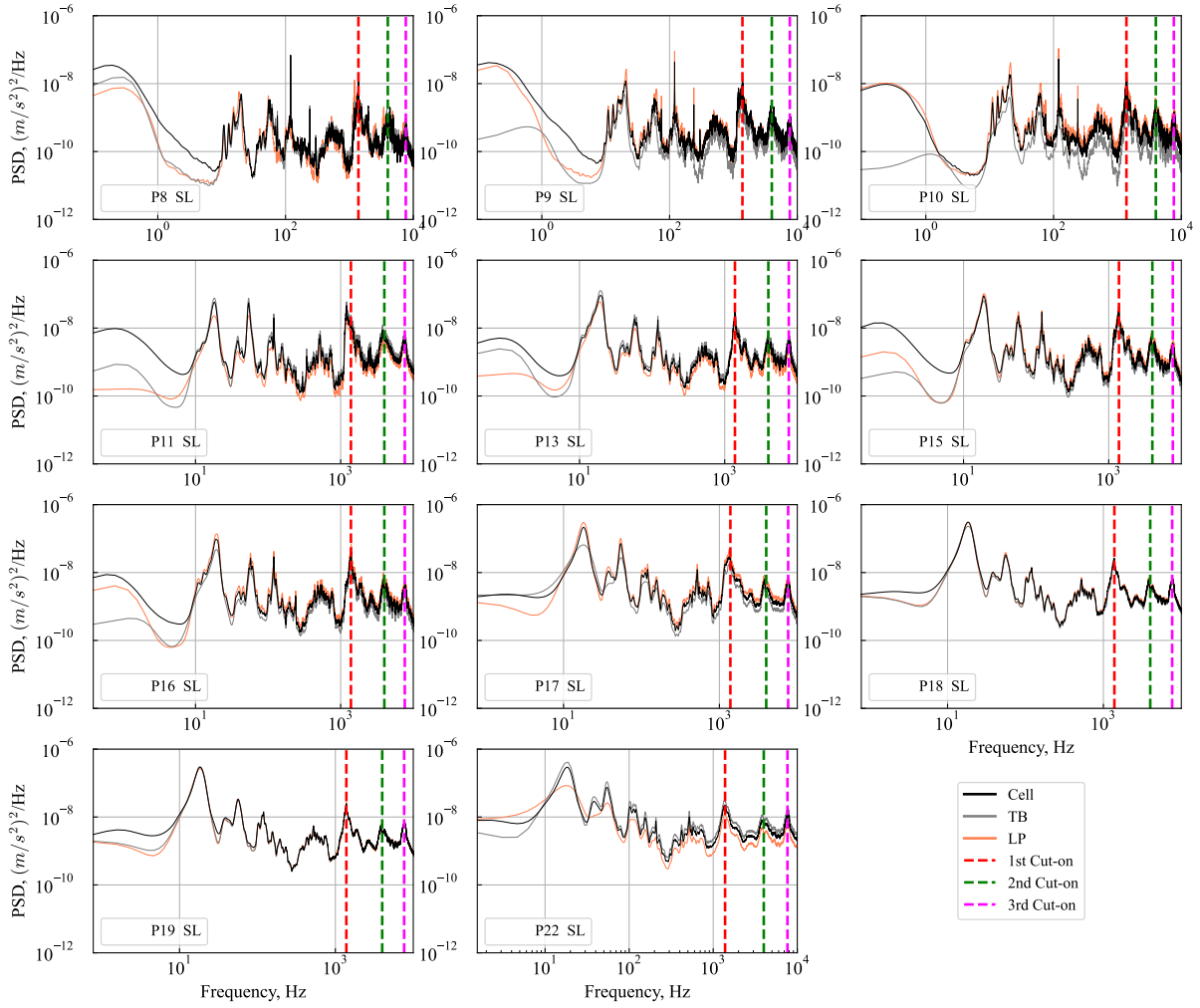


Figure 6.14 – Power Spectral Density (PSD) of the acceleration measurement at the liquid piston (LP), Taylor bubble (TB) and unit cell (cell) for all experimental points in the slug pattern. The dashed vertical lines indicate the three lowest cut-on frequencies.

The first three peaks at the acceleration PSD, around 20 Hz, 60 Hz and 105 Hz, are related to the resonance frequency of the lowest bending modes of the pipe, while the large amplitude peak just after 1 kHz is associated to and increase in the pipe modal density due to the $n = 2$ circumferential mode cutting-on, as shown in Table 5, red dashed line in Figure 6.14. Similarly, there is a sharp increase in the vibration amplitude at frequency bands closer to the subsequent $n = 3$ and $n = 4$ circumferential wave modes, at 4 kHz and 7.5 kHz. This is accompanied by slight amplitude increases at the corresponding frequencies of the pressure PSD, Figure 6.13. At these frequency bands, a coupling between the pressure and structure waves are expected, as shown for a single-phase flow by Kirby and Duran (KIRBY; DUAN, 2019). This is caused by the wave mode veering, in which the kinetic energy of both pressure and structural mode are equally distributed, due to a gyroscopic coupling (MACE; MANCONI, 2012). This effect explains the experimentally observed vibration amplification at high frequencies found by Carvalho et al. (CARVALHO; FIGUEIREDO; SERPA, 2019) and it is further investigated by a classical Single-Input/Single-Output (SISO) system estimator, assuming the pressure as

input and the acceleration as output.

Figure 6.15 shows the Frequency Response Function (FRF) amplitude from the H_T estimator and the coherence function $\gamma(\omega)$ for the liquid piston (LP), the Taylor Bubble (TB) and the unit cell (Cell) at the experimental points 13 and 17. Unlike the classical H_1 and H_2 estimators, the H_T assumes the same level of additive and uncorrelated measurement noise in both input and output (SHIN; HAMMOND, 2008). The coherence function gives the level of linear relation between two signals and it equals unity for linearly related signals and zero for uncorrelated signals. In the context of the H_T estimator, the coherence function can be smaller than one in case there are nonlinearity in the system, measurement noise and/or more than one input for the same output. Assuming the pipe is operating in the linear regime, which is reasonable given the low-pressure levels, and that the measurement noise is averaged out due to the significantly high number of segments in the 600 seconds of measurement, the values of the coherence function close to zero can be associated to more than one input to the acceleration response. This is expected due to the spatial coherence induced by Turbulent Boundary Layer (TBL) excitation (MAXIT; KARIMI; GUASCH, 2021). Remarkably, the coherence levels increase sharply at the cut-on frequencies and at a frequency band just below 1 kHz. Investigating the latter is outside the scope of this work. This highlights the localised nature of the coupling. In the following section, this coupling is more closely investigated.

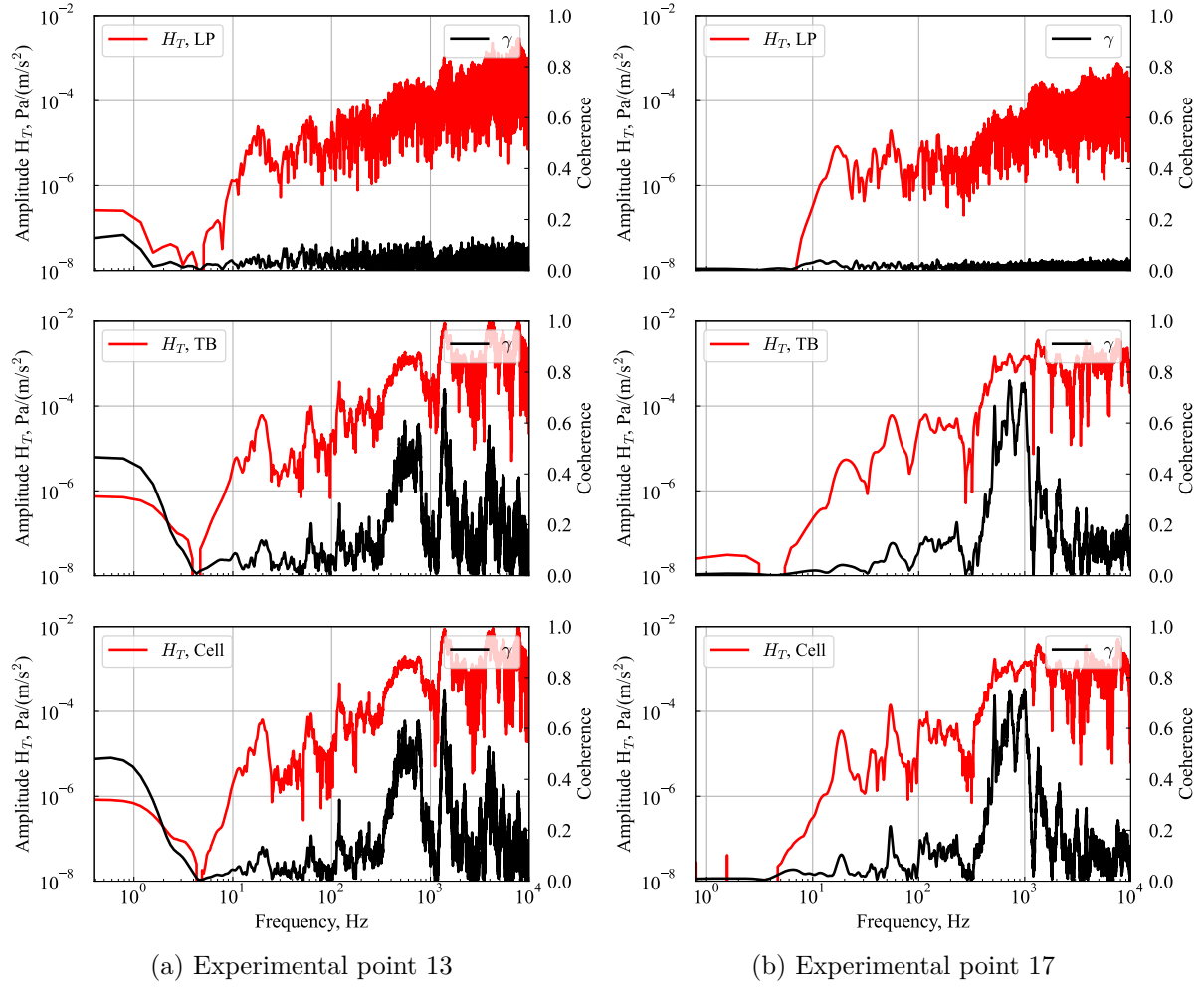


Figure 6.15 – Amplitude of the estimated Frequency Response Function (FRF) and the coherence (γ) at the liquid piston (LP), Taylor bubble (TB) and the unit cell (Cell) for experimental points (a) 13 and (b) 17.

6.2.3 Structural response at the cut-on band

In this section, the vibration response at the lowest cut-on circumferential wave mode and its relation with the void fraction is investigated. Figure 6.16 shows the acceleration PSD for both experimental points 13 and 17, highlighting the three lowest cut-on frequencies of the circumferential wave modes. It also shows the filtered acceleration PSD, with a zero-phase pass-band Butterworth filter, of order 2 and central frequency at f_{n2} with 30 % bandwidth. Figure 6.17 presents 15 seconds of the time domain acceleration measurement and the corresponding synchronised void fraction along with the filtered acceleration, also for both point 13 and 17. The filtered acceleration has much lower amplitude than the full acceleration, as expected, and it is possible to notice that there are several transient responses at the time when there is an abrupt change of void fraction. This is associated to the passage of the Taylor bubble. Figure 6.18 shows the time domain envelope of filtered acceleration along with the synchronised void fraction for only 5 seconds. The envelope is calculated using the amplitude of the analytical signal obtained

from a Hilbert transform. It is possible to notice that, in addition to the acceleration transients, the vibration amplitude is modulated by the passage of the Taylor bubble. It tends to increase for increasing void fraction, i.e., at the passage of a Taylor bubble. This modulation in the response is also associated to the number of bubbles, its average diameter and its frequency content, as shown in Section 6.2.1.

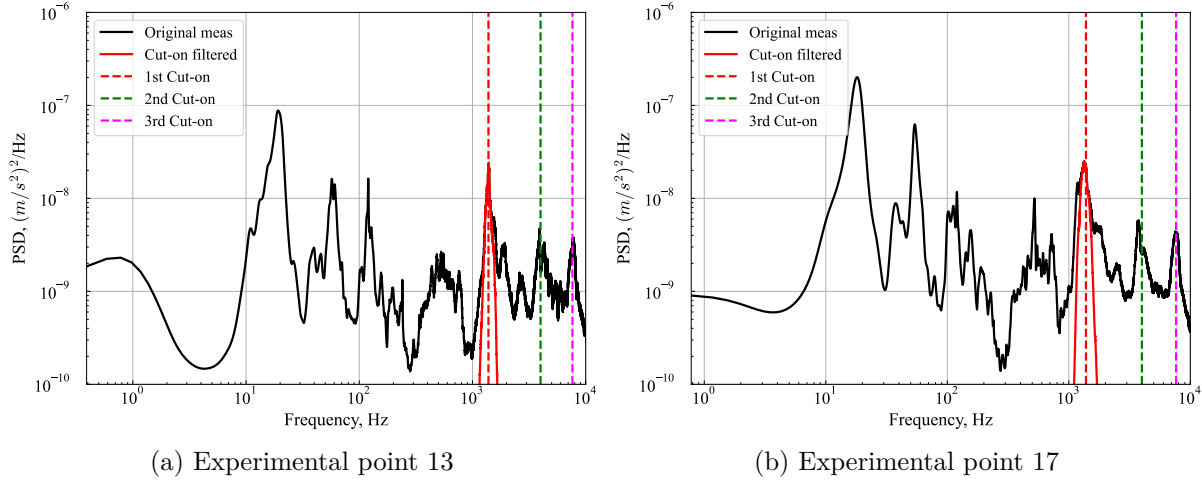


Figure 6.16 – Power Spectral Density (PSD) of the full acceleration measurement and filtered around the lowest cut-on frequency for experimental points (a) 13 and (b) 17. The dashed vertical lines indicate the three lowest cut-on frequencies.

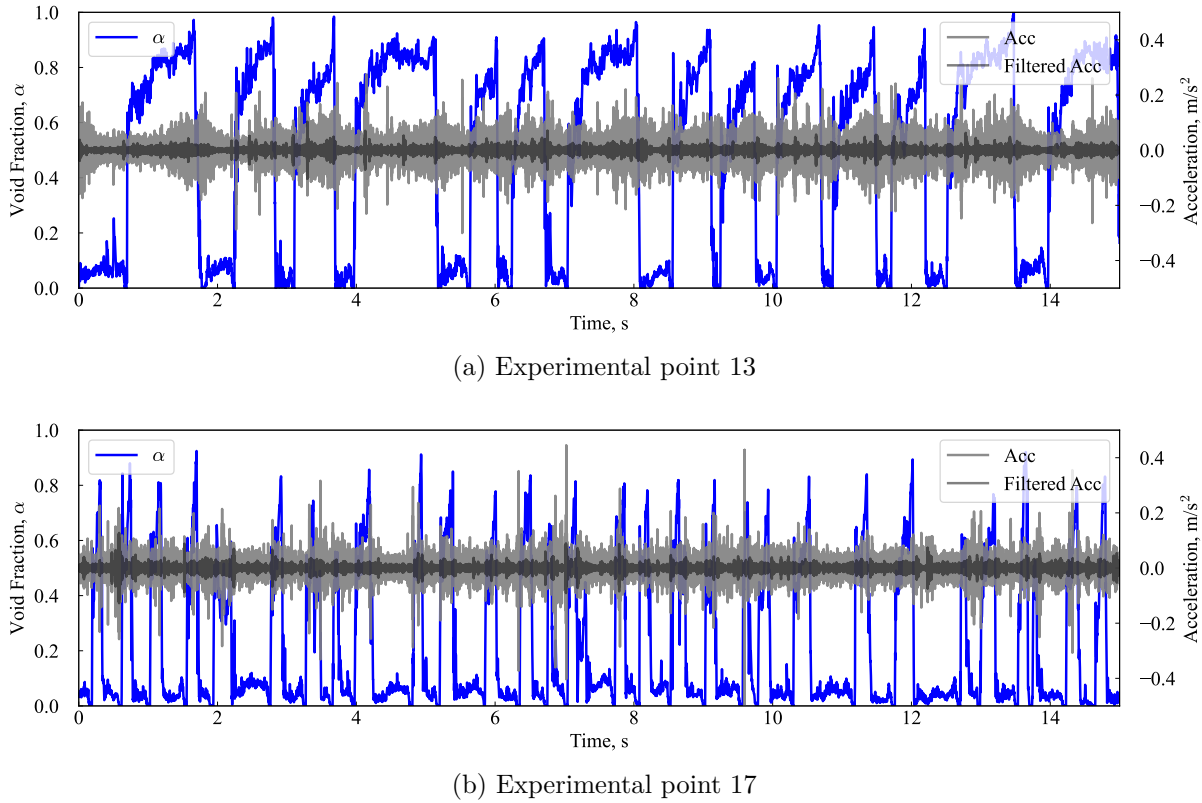
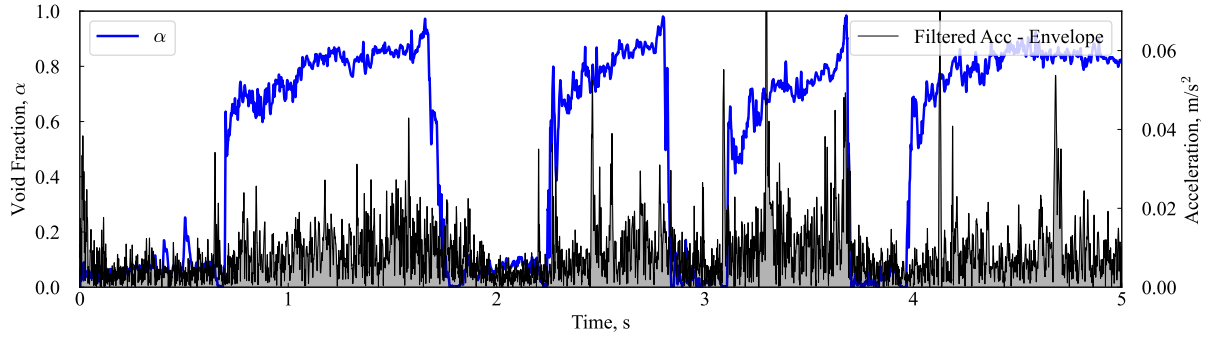
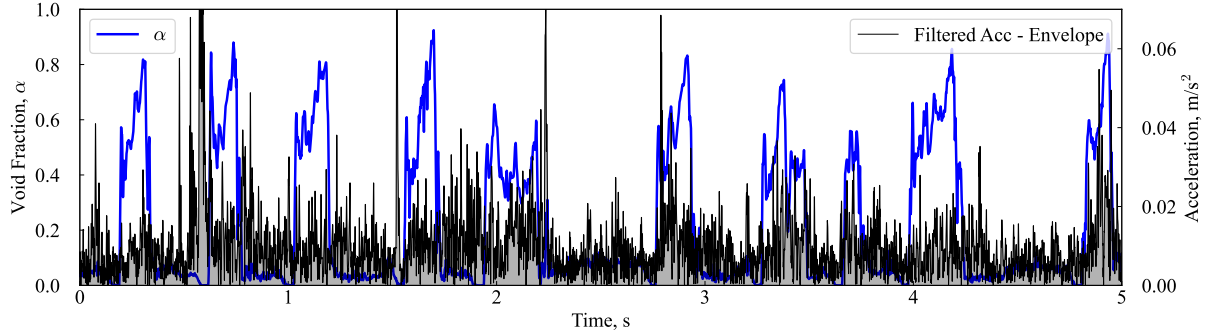


Figure 6.17 – Time domain acceleration measurement (gray line), filtered around the lowest cut-on frequency (black line) along with the synchronised void fraction (blue line) for experimental points (a) 13 and (b) 17.



(a) Experimental point 13



(b) Experimental point 17

Figure 6.18 – Time domain envelope (black line) of the filtered signal along with the synchronised void fraction (blue line) for experimental points (a) 13 and (b) 17.

Figure 6.19 shows the PSD amplitude of the void fraction and the acceleration envelope at the lowest cut-on frequency for both experimental points. Unlike the previous PSD, the size of the segments in Welch’s approach are chosen to be at least two times longer than the largest unit cell for a suitable frequency resolution. The demodulation of the acceleration amplitude from the Hilbert transform shows that the main frequency content of vibration response matches that of the void fraction series for both cases. It can be shown that the peak of the void fraction PSD is a direct measure of the frequency of passage of the unit cell and that the PSD shape can be modelled as high-order two-states Markov chain (VIEIRA et al., 2023). Remarkably, the acceleration PSD has preserved this format for lower frequencies, despite of the nonlinear relation between the series.

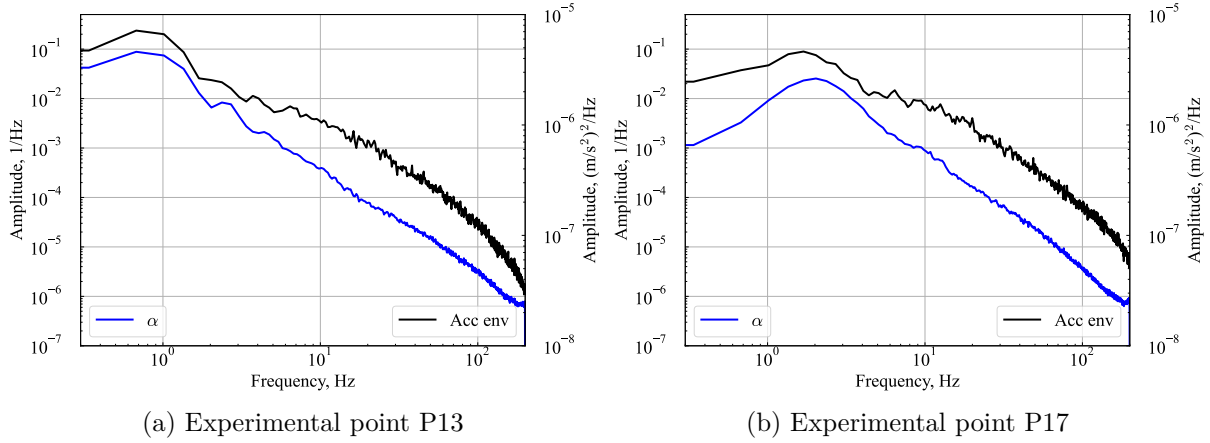


Figure 6.19 – Power Spectral Density (PSD) amplitude of the void fraction (blue line) and the acceleration envelope at the lowest cut-on frequency (black line) for experimental points (a) 13 and (b) 17.

6.3 Slug flow characterisation

In this section, the translational velocity (V_{TB}) and the slug frequency (f_{slug}) parameters are estimated for the slug pattern points from pressure and acceleration measurements only, and finally, a flow pattern classification is performed for all experimental points using a single sensor, based on the phenomenon of fluid-structure coupling.

6.3.1 Slug frequency

In this section, the number of slug units that passed a fixed point on the pipe during a specified time (GREGORY; SCOTT, 1969), i.e., the slug frequency (f_{slug}), is estimated from the demodulated acceleration signal. Like the other characteristics of the slug pattern, the f_{slug} is an essential parameter that in some cases can cause a series of operational problems, when in high frequency (HILL et al., 1996). Furthermore, it is a critical issue in the development of slug flow models (MOHMMED; AL-KAYIEM; OSMAN, 2021). Measurement from the Ac_1 was used and the demodulation was performed as discussed in the previous section, i.e., by filtering the signal at the lowest cut-on frequency (f_{n2}) with a zero-phase filter and then calculating the signal envelope using the Hilbert transform.

Figure 6.20 shows the auto PSD of two typical measurement points, P10 (a) and P13 (b), estimated by using a Welch's segment and average approach (SHIN; HAMMOND, 2008) using various number of segments N_{seg} . The number of blocks was increasing for $N_{seg} = 5$, $N_{seg} = 10$, and $N_{seg} = 100$. The low frequency range is related to mass transport, i.e. slug frequency, and they appear as distinct sharp peaks. From this frequency, the PSD amplitude decreases sharply towards higher frequencies. As expected, when the segment number increases the averaging approach has the effect of smoothing the amplitude of the

PSD. This highlights the importance of adopting an adequate N_{seg} , so that this frequency peak is well defined rather than smoothed out, which requires an a priori estimation of f_{slug} , which can be done by using a model, as shown in section 2.5.1.

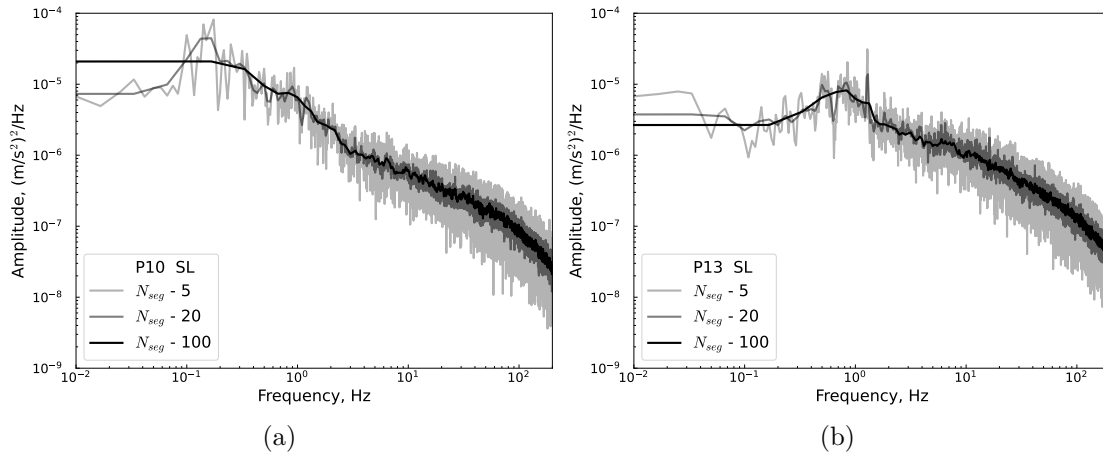


Figure 6.20 – Power Spectral Density (PSD) of the acceleration envelope at the lowest cut-on frequency for point experimental P10 (a) and P13 (b) with different numbers of blocks: $N_{seg} = 5$, $N_{seg} = 50$, and $N_{seg} = 100$.

Figure 6.21 presents the auto PSD of the demodulated acceleration signal (black lines) and the auto PSD of the corresponding void fraction measurement α (blue lines) for all of the slug flow points in the test matrix. It also shows the frequency at the PSD maximum value of the demodulated vibration measurements (dashed red vertical lines). The demodulated accelerometer measurements were performed by filtering the signal at the lowest cut-on frequency f_{n2} . For both cases, the PSDs were calculated using the Welch's method, with a number of blocks $N_{seg} = 20$ and a Hanning window. The void fraction obtained from the conductance-based meter is a direct measure of the mass wave in the flow (SANTOS et al., 2019). Assuming the slug flow as a stochastic process, it can be shown that the peak of the void fraction PSD is a direct measure of the most probable frequency of passage of the unit cell and that the PSD decay rate at higher frequencies is related to the short term variability in the slug unit cell (VIEIRA et al., 2023). It can be seen that the demodulation of the acceleration amplitude shows that the main frequency content of vibration response matches that of the void fraction series for all cases. Figure 6.22 presents the estimated f_{slug} from the acceleration compared to those obtained from the void fraction, note that for most of the experimental points, the difference between the two approaches is below 15%. Table 6 presents in more detail the f_{slug} value and the relative error calculated for each experimental point.

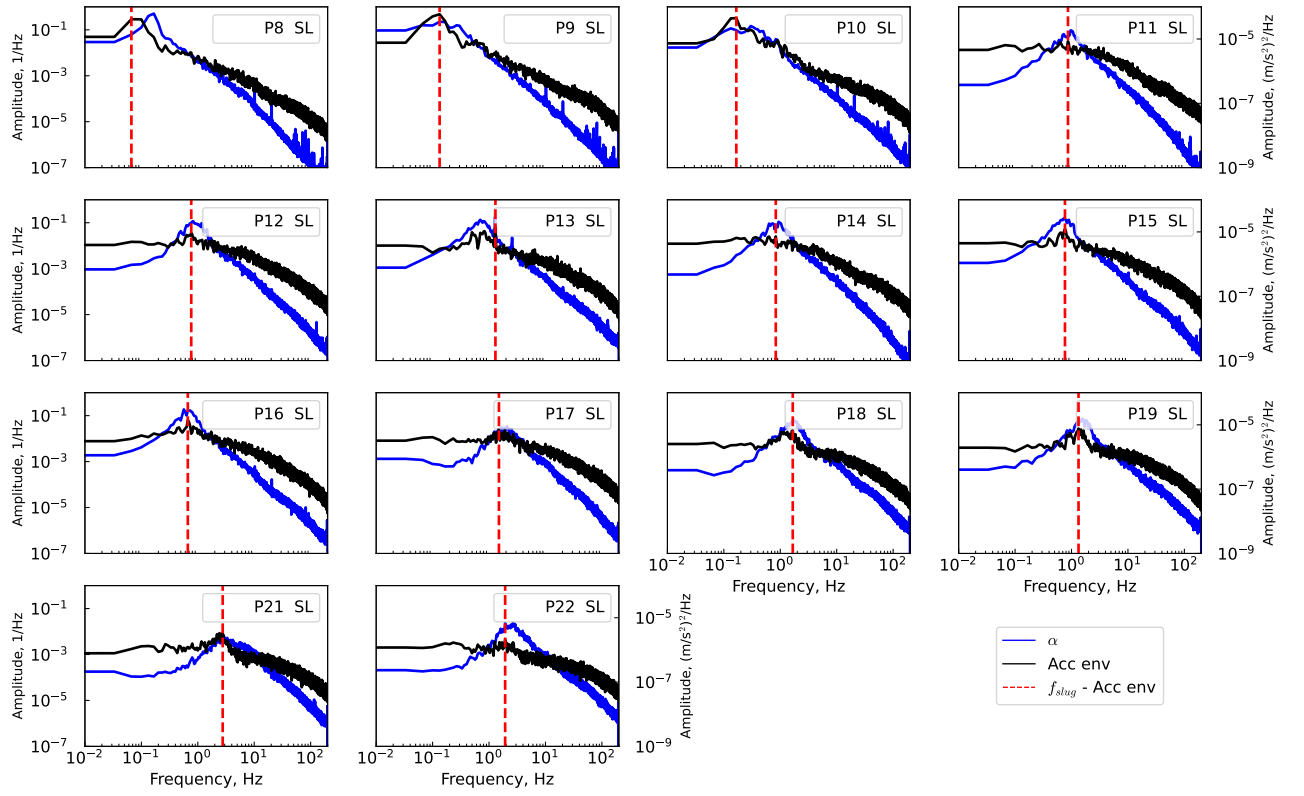


Figure 6.21 – Power Spectral Density (PSD) of the void fraction signals (blue line) and the acceleration envelope at the lowest cut-on frequency (black line) for each slug experimental points.

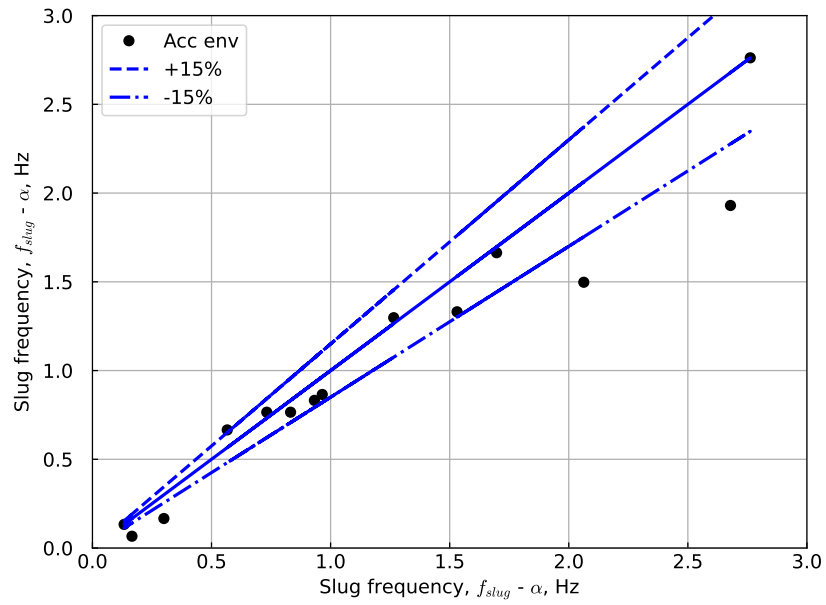


Figure 6.22 – Slug frequency estimation from the demodulated acceleration measurement $f_{slug} - \text{Acc env}$ (black dots) and the void fraction signals $f_{slug} - \alpha$ (blue line), for each slug experimental points.

Tabela 6 – Estimates of the slug frequency (f_{slug}) obtained by the acceleration sensors (Acc env) and those obtained by the void fraction signal (α), the relative error are shown.

Point	J_{sl} , m/s	J_{sg} , m/s	$f_{slug} - \alpha$, Hz	$f_{slug} - \text{Acc env}$, Hz	error, %
8	0.580	0.504	0.166	0.067	59.905
9	0.589	0.977	0.133	0.133	0.084
10	0.578	1.429	0.300	0.166	44.537
11	1.191	0.382	0.965	0.865	10.339
12	1.315	0.845	0.832	0.765	8.006
13	1.353	1.400	1.265	1.298	2.596
14	1.300	0.493	0.932	0.832	10.735
15	1.242	1.001	0.732	0.765	4.562
16	1.212	1.218	0.566	0.666	17.590
17	2.150	0.532	2.063	1.498	27.411
18	2.044	1.017	1.697	1.664	1.951
19	2.017	1.182	1.531	1.331	13.056
21	3.042	0.421	2.762	2.762	2.762
22	2.927	0.854	2.679	1.930	27.954

6.3.2 Translational velocity

In this section, an approach for estimating the slug velocity from two accelerometers is proposed. The goal is to estimate the time delay corresponding to the peak of the cross-correlation function between the signals from two nearby sensors. With the calculated time delay, it is possible to use the known distance between the sensors to estimate the velocity. Figures 6.23 (a) and (b) present the Basic Cross-Correlation (BCC) between the demodulated acceleration measurements for experimental points 10 and 13, using accelerometers Ac_1 and Ac_5 . In Figure 6.23 (b) it is possible to observe that instead of one clear peak, the BCC function presents two, the lowest in the value of $\tau_{x_1x_2} = 0.0025s$ and another in the value of $\tau_{x_1x_2} = 0.2645s$, with estimated speeds of 325.079 m/s and 3.023 m/s, respectively. The second velocity corresponds to the order of magnitude typical to Taylor bubble velocity (V_{TB}). The first peak is possibly associated with the velocity of a structural wave, which is the subject of further investigation. On the other hand, for experimental point P10, Figure 6.23(a), the second peak is not as evident, the same phenomenon was observed for all low f_{slug} points, making it necessary to apply the Generalised Cross-Correlation (GCC) techniques to highlight this delay corresponding to low-velocity phenomena.

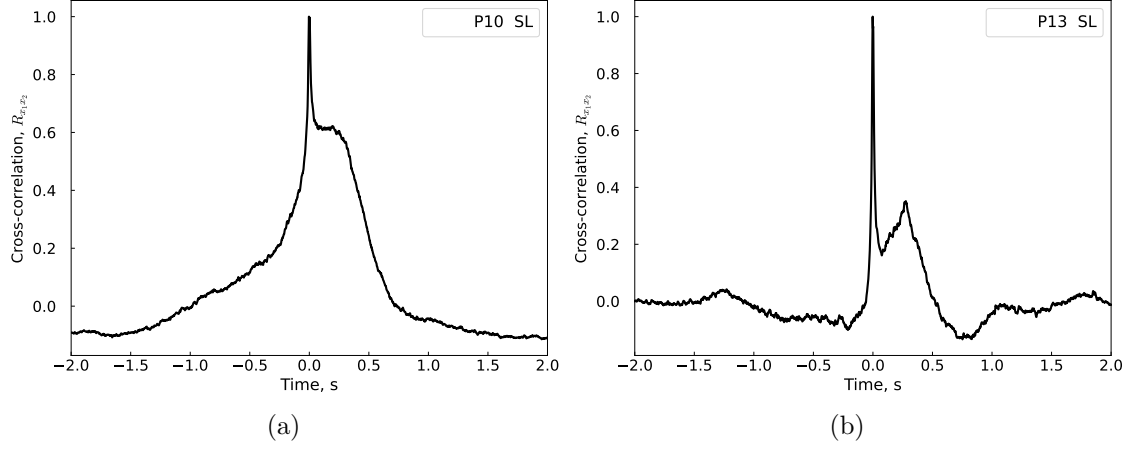


Figure 6.23 – Basic Cross-Correlation (BCC) of the two demodulated acceleration measurements Ac_1 and Ac_5 , for experimental points P10 (a) and P13 (b).

Figure 6.24 presents the Generalised Cross-Correlation (GCC) between the demodulated acceleration measurements. Note that the red dashed lines indicate the point where the time delay was detected as a local maximum. In order to highlight the second peak of the cross-correlation function, corresponding to V_{TB} , a Maximum Likelihood (ML) weighting function was applied, Section 4.2.6, which highlights the region of high coherence. Note that at points P8 and P11, the peak and maximum are not so evident when compared to the other cases. This is due to the low slug frequency and consequently a low number of unit cells during the measurement time, as discussed in the previous section. On the other hand, note that for points where the passing frequencies are high, the peak is very pronounced, such as points P13, P21, and P22. Figure 6.25 shows the estimated velocity for all slug flow patterns from the test matrix compared to the estimates from the void fraction measurement, with error lines of 15%. Note that all measurements are within the error range indicating a good agreement between the measured and expected results. Table 7 presents in more detail the f_{slug} value and the relative error calculated for each experimental point.

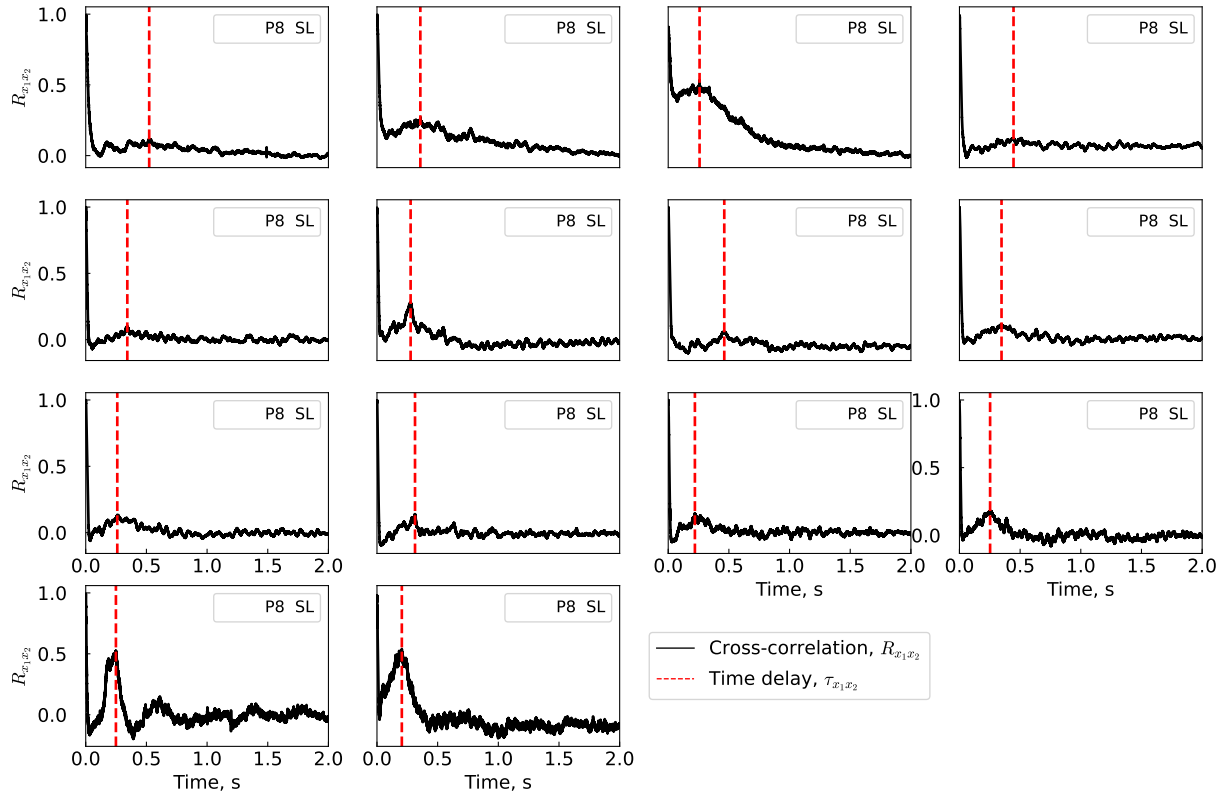


Figure 6.24 – Generalized Cross-Correlation (GCC) of the two demodulated acceleration signals Ac_1 and Ac_5 , for each slug's experimental point. The red dashed lines correspond to the maximum amplitude peaks of the GCC function.

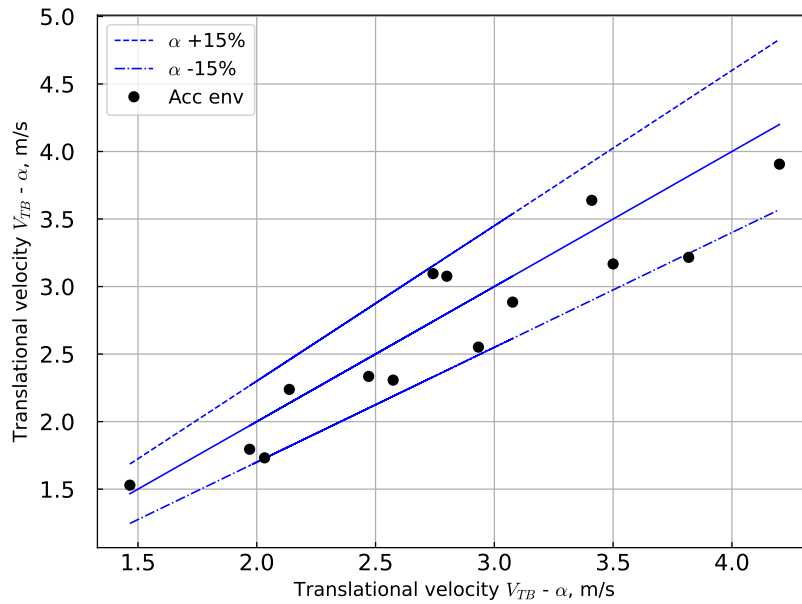


Figure 6.25 – Translational velocity estimation from the demodulated acceleration measurement V_{TB} - Acc env (black dots) and the void fraction measurement V_{TB} - α , for each slug experimental point.

Tabela 7 – Estimates of the translational Velocity (V_{TB}) obtained by the acceleration sensors (Acc env) and those obtained by the void fraction signal (α), the relative error are shown.

Point	J_{sl} , m/s	J_{sg} , m/s	$V_{TB} - \alpha$, m/s	$V_{TB} - \text{Acc env}$, m/s	error, %
8	0.580	0.504	1.466	1.529	4.331
9	0.589	0.977	2.137	2.238	4.749
10	0.578	1.429	2.742	3.096	12.893
11	1.191	0.382	1.970	1.795	8.872
12	1.315	0.845	2.471	2.335	5.516
13	1.353	1.400	3.077	2.885	6.243
14	1.300	0.493	2.033	1.732	14.824
15	1.242	1.001	2.574	2.307	10.370
16	1.212	1.218	2.800	3.076	9.874
17	2.150	0.532	2.933	2.552	13.000
18	2.044	1.017	3.410	3.638	6.695
19	2.017	1.182	3.500	3.167	9.505
21	3.042	0.421	3.818	3.216	15.765
22	2.927	0.854	4.200	3.906	4.200

6.4 System identification

In this section, a system identification analysis using pressure, pressure gradient signal and acceleration signals will be presented for all slug experimental points. All signals were downsampled to a frequency of 50 Hz. For the acceleration signals, the envelope of the signal downsampled at 50 Hz was used, capturing only the first structural flexural mode. The pressure gradient signal along the pipe cross-section is obtained from the difference between sensors P_{re1} and P_{re6} , which are located at the same measurement point but in opposite directions, as shown in Figure 5.2. This approach aims at reducing flow disturbances in the pressure signal that propagates at the velocity of sound.

6.4.1 Delay time

As discussed in Section 4.4.2, two methods stand out in determining the delay time (τ): the Autocorrelation Function (ACF) and the Average Mutual Information (AMI) method. Figure 6.26 presents the AMI (continuous lines) and ACF (dashed lines) functions obtained from pressure sensor P_{re10} , pressure gradient from sensors $\Delta(P_{re1} - P_{re6})$, and acceleration sensor Ac_1 for experimental point P13. The delay time is found when the ACF function reaches the zero point for the first time, characterising linear independence between the signal and its delayed version. This value is marked with the vertical dashed line in the figures. In general, the smaller the AMI, the more independent the shifted time series and the more additional information a surrogate series can provide. For this reason, Fraser e Swinney (1986) proposed using the position of the first minimum as the optimal

value of τ . Thus, the first local minimum is highlighted with a vertical continuous line, and its value is shown in the legend.

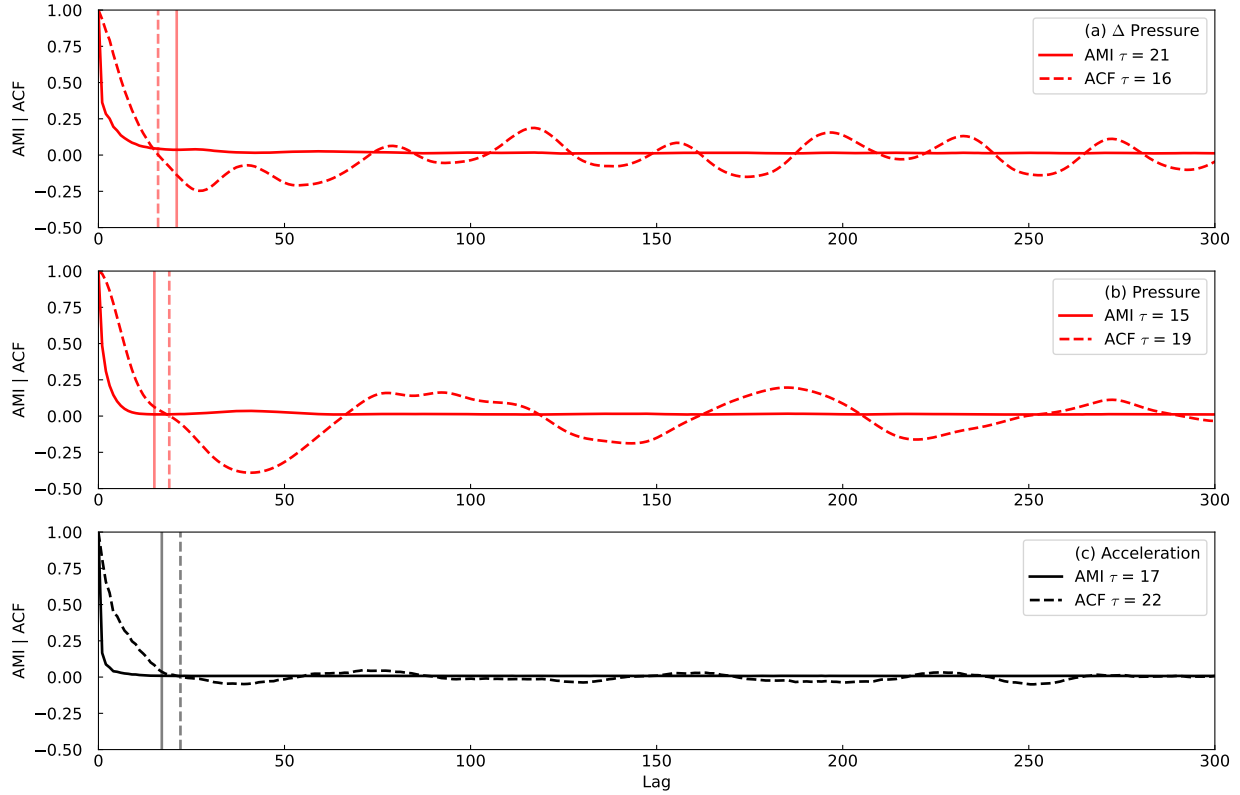


Figure 6.26 – Time delay estimated from Autocorrelation Function (ACF) and Average Mutual Information (AMI) for pressure gradient (a), pressure (b), and acceleration signal (c) for experimental point P13. The vertical lines on the ACF and AMI curves indicate the first zero-crossing and the first local minimum.

Note that in all three signals, the ACF exhibits periodic characteristics, indicating that there are repeating patterns in how the time series values correlate with themselves over time. The reason for this is the oscillatory nature of the signal in the intermittent pattern, as discussed earlier. The choice of methods for estimating time delay is crucial to capture the system's dynamics. The first zero-crossing of the ACF is more suitable for linear relations, while the first local minimum of AMI is more sensitive to non-linear and complex relations.

6.4.2 Embedding system

The embedded dimension (d_e) is another important parameter for state space reconstruction, which is not the dimension of the system itself, but the number of dimensions needed to correctly embed the attractor in the phase space. As presented in Section 4.4.1, the choice of its value is defined by the Whitney incorporation theorem of $d_e \geq 2d_a + 1$, where d_a is the original dimension of the attractor. The first method is known as the False Neighbors Neighbor (FNN) method. In this method, the projection of the system

in a specific dimension is used to determine whether a neighbour is "true" or "false" by comparing it to its projection in a higher dimension. The minimum dimension is determined when the distance between neighbours stops growing, indicating that there are no more false neighbours. Figure 6.27 presents the results of the percentages of false neighbours concerning the increase in the embedding dimension, represented by continuous lines. The ideal value of d_e is considered achieved when the number of false neighbours drops to 10% (SAVI, 2006).

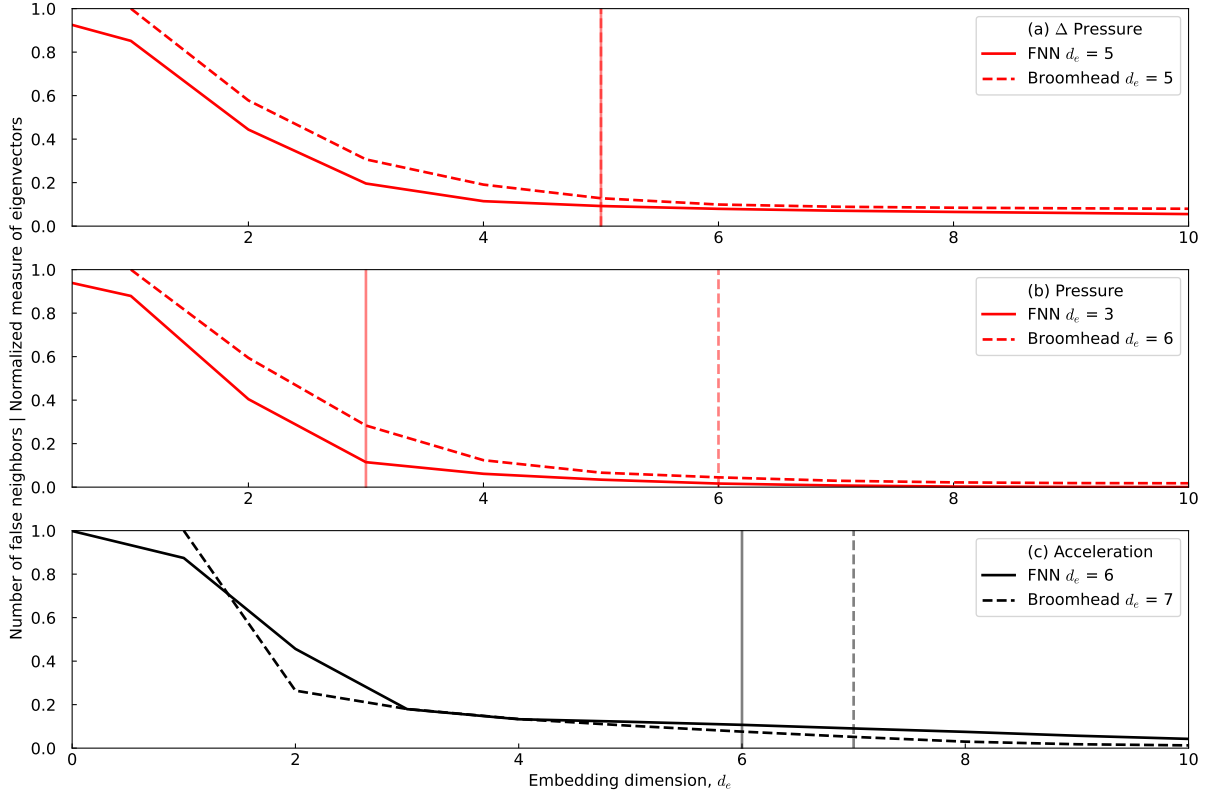


Figura 6.27 – Embedding dimension estimated from FNN (continuous line) and Broomhead (dashed line) method for pressure gradient (a), pressure (b), and acceleration signal (c) for experimental point P13. The vertical lines indicate the estimated embedding dimension for each method.

The second method was proposed by Broomhead e King (1986), the main feature is to obtain a noiseless unfolded attractor considering only the main singular value and disregarding irrelevant high-order dynamics related to lower singular values. The criterion used to define the threshold is applied by Equation 4.42. Figure 6.27 shows the curve of each singular value in relation to the total (continuous lines), calculated from the diagonal of the Hankel matrix of the data. This allows us to observe the normalized measure of the importance of each dimension, where values close to 1 indicate that the dimension is highly relevant for explaining the total variation, while values close to 0 indicate that the dimension has little relevance.

Broomhead's method identifies the most significant directions in the data and performs dimensionality reduction. Meanwhile, the FNN method focuses on identifying

recurrence relationships in the data, is highly dependent on the accurate estimation of time delay, and is sensitive to the presence of noise in the time series. In summary, the Broomhead method is more robust regarding system dimension estimation. For most points, d_e has values between 5 and 7, Figure 6.27, according to Whitney incorporation theorem, resulting in a system of order $d_a = 3$. Despite being three signals of different nature, the dimensions of the system are the same, as they are part of the same dynamical system. This result is in agreement with the work developed by Vieira (2022) in which, based on signs and void fraction, the author estimated a dimension of $d_a = 3$ using the Broomhead method. Despite the embedding dimension not being a requirement for the existence of chaos in a dynamical system, continuous chaotic systems must have at least three dimensions in the state space Savi (2006).

6.4.3 State space reconstruction

Introduced by Takens analysis theorem, which allows the reconstruction of a d_e -dimensional state space similar to the original d_a -dimensional state space from a single measured variable (TAKENS, 1981). The reconstructed state space exhibits a smooth variation of coordinates compared to the original space, preserving important invariant geometric properties such as fixed point eigenvectors, fractal dimension of the attractor, and Lyapunov exponent of trajectories (CASDAGLI et al., 1991). Figures 6.28, 6.29 and 6.30 show 10 seconds of time series of pressure gradient, pressure and acceleration and the reconstructed state space, respectively, for experimental point P13 in the slug pattern.

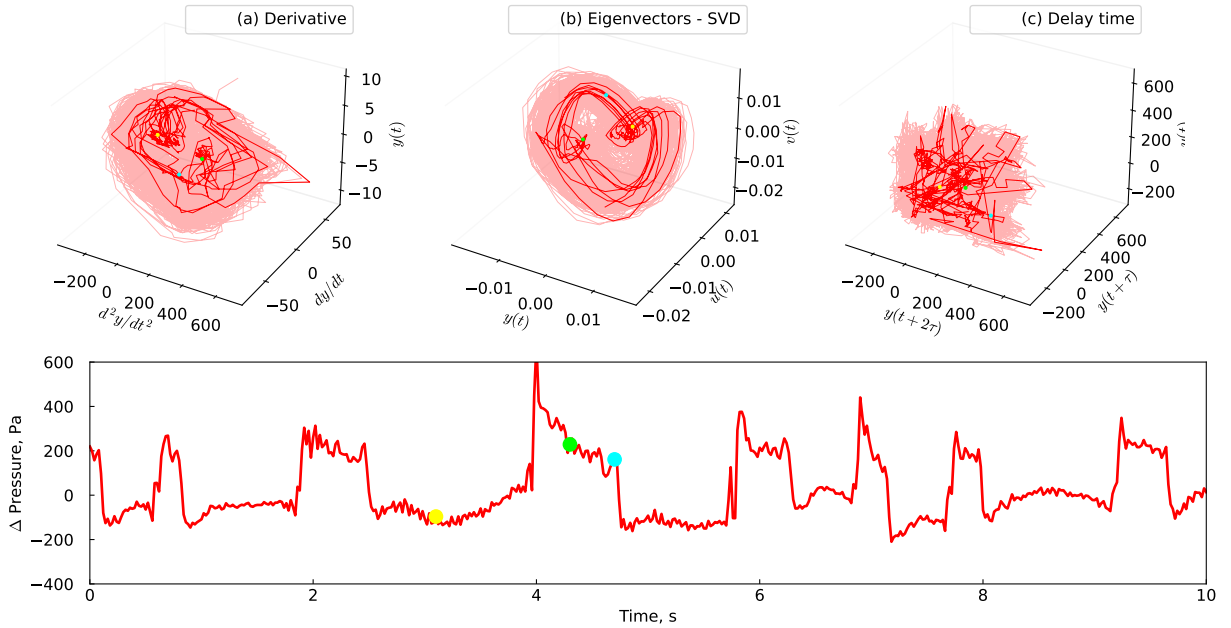


Figure 6.28 – Time series of pressure gradient signal and reconstructed state space using the techniques of derivatives (a), singular value decomposition (b), and time delay (c) for experimental point P13. Colored circles mark points in the time series and state spaces.

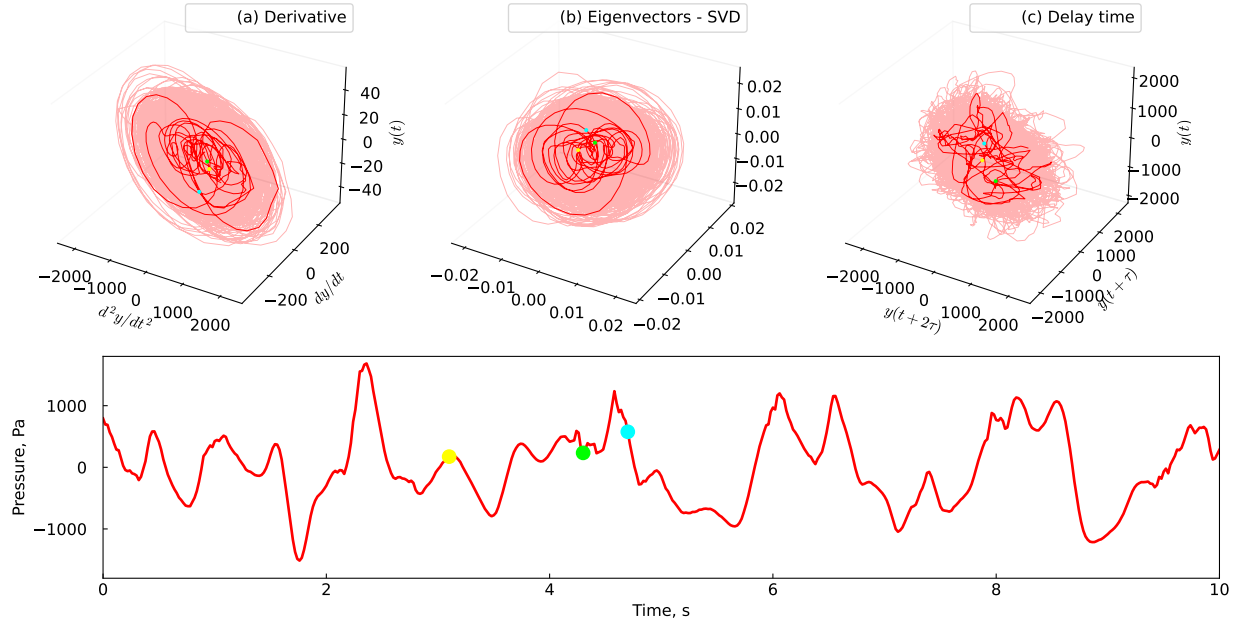


Figure 6.29 – Time series of pressure signal and reconstructed state space using the techniques of derivatives (a), singular value decomposition (b), and time delay (c) for experimental point P13. Colored circles mark points in the time series and state spaces.

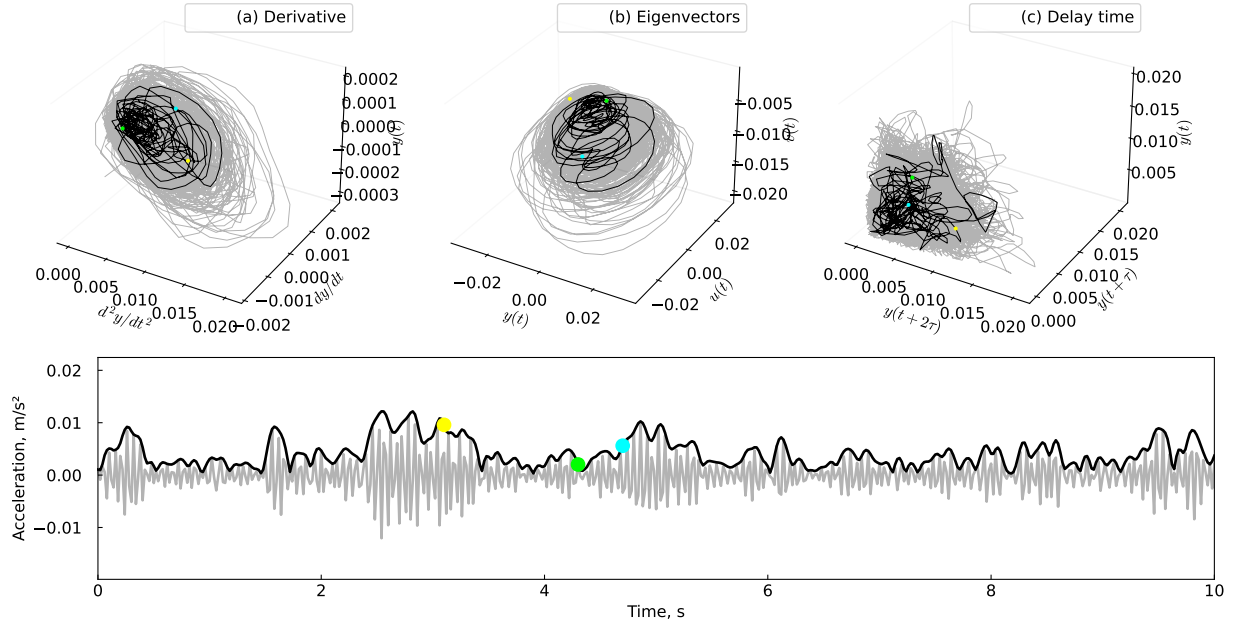


Figure 6.30 – Time series of acceleration signal and reconstructed state space using the techniques of derivatives (a), singular value decomposition (b), and time delay (c) for experimental point P13. Colored circles mark points in the time series and state spaces.

The first method is the derivative method, in which the coordinates are numerical approximations of the successively higher-order derivatives of the measured variable (PACKARD et al., 1980). Note that in Figure 6.28 (a), which shows the reconstructed state space using the method of derivatives, the state space exhibits two clear attractors

varying between the two regions of the Taylor bubble and liquid piston, indicated by the coloured circles in the time series and in the state space. The derivatives were calculated using the Savitzky–Golay filter (SAVITZKY; GOLAY, 1964). This is in agreement with the results presented in the work of Vieira (2022), where the author analysed the state space based on the void fraction signals for intermittent flows and also presented two fixed points varying between the liquid piston and the Taylor bubble. On the other hand, in Figure 6.29 (a), when only a single time series from a single pressure sensor is used, the state space becomes dense and complex, indicating that the system may be more chaotic or less structured in this representation, as well as using the acceleration signals, as shown in Figure 6.30 (a).

The second method employs the technique of Singular Value Decomposition (SVD) to identify directions in the embedding space that can be ordered based on the magnitude of trajectory projections. This results in the creation of a new coordinate system. The eigenvectors are then used to construct the coordinates in the reconstructed state space. It is worth noting that these coordinates are optimal in the sense of minimising the distortion of the information present in the original time series (BROOMHEAD; KING, 1986). Therefore, it can be observed in Figures 6.28 (b), 6.29 (b), and 6.30 (b) that a smoothed version of the reconstructed state space is presented, and it is worth noting that, similar to the results using derivatives, the state space oscillates between two distinct fixed points.

The last method employs the time delay embedding technique to reconstruct the state space by shifting the time series by an appropriate number of time steps. In all reconstructed state spaces, represented in Figures 6.28 (c), 6.29 (c), and 6.30 (c), they exhibit a more regular structure compared to other results. This occurs because the delay coordinates are essentially time-shifted copies of the original time series. On the other hand, the SVD-based technique seeks to identify the most significant directions or principal modes of variation in the data resulting in a smoother and simplified representation of the state space. This can yield trajectories with a more natural and continuous appearance, proving particularly effective in capturing the underlying structure of highly nonlinear systems.

6.4.4 Largest Lyapunov exponent

Despite deterministic temporal evolution, chaos is characterised by its unpredictability. As a result of the inherent instability in solutions, this unpredictability is reflected in what is referred to as sensitive dependence on initial conditions. For example, orbits in state space can be thought of as parametric functions of a variable that is something like time. Take one of these orbits as a reference, so the paths between the two orbits will also be a function of time. In predominantly periodic systems, these trajectories diverge over time very slowly, while in chaotic systems, this separation is much faster. Thus, the

average exponent of this increase quantifies the force of chaos in the dynamic system, known as the Lyapunov exponent. Chaotic systems are characterised by the exponential divergence of close orbits in state space. The Lyapunov exponent must be positive for a low-dimensional deterministic process, zero for a linear process, and infinite for a stochastic process (WOLF et al., 1985b; Pesin, 1977).

The largest Lyapunov exponent was determined using the algorithm proposed by Kantz (1994) and Rosenstein, Collins e De Luca (1993), as presented in Section 4.4.5. The Lyapunov exponent was estimated using 600 seconds of the pressure time series, the τ provided by the AMI and d_e estimated by Broomhead e King (1986) method. Figure 6.31 shows the curve $S(k)$ versus k from the acceleration, pressure gradient, and pressure calculated through Equation 6.26, for the reconstructed state space using the consecutive delay time method (a) and using the three main components resulting from the decomposition of singular values (b). Using the first method, it is possible to observe that the curves present short-term oscillations with a slowly increasing amplitude. When the amplitude reaches a critical value, it breaks down and the oscillations start again. As observed by Kantz e Schreiber (2003), the oscillations exhibit a clear origin in the periodicity of the signal, which is consistent with the periodic nature of the slug pattern, characterised by periodic oscillations between the liquid piston and the Taylor bubble, as previously discussed. Similar behaviour was found by (VIEIRA, 2022) for void fraction time series. This intrinsic characteristic of the input signal results in periodic oscillations in behaviour, which, in turn, can compromise the accuracy of the Lyapunov exponent measurement due to instabilities in the resulting curve. On the other hand, when using the principal modes, Figure 6.31 (b), the curve tends to exhibit significant smoothing, thus mitigating the previously observed instabilities. Smoothing occurs because the principal modes of variation tend to capture general trends and underlying patterns in the data, while the less important modes of variation (which are discarded) are often related to noise or random fluctuations in the data.

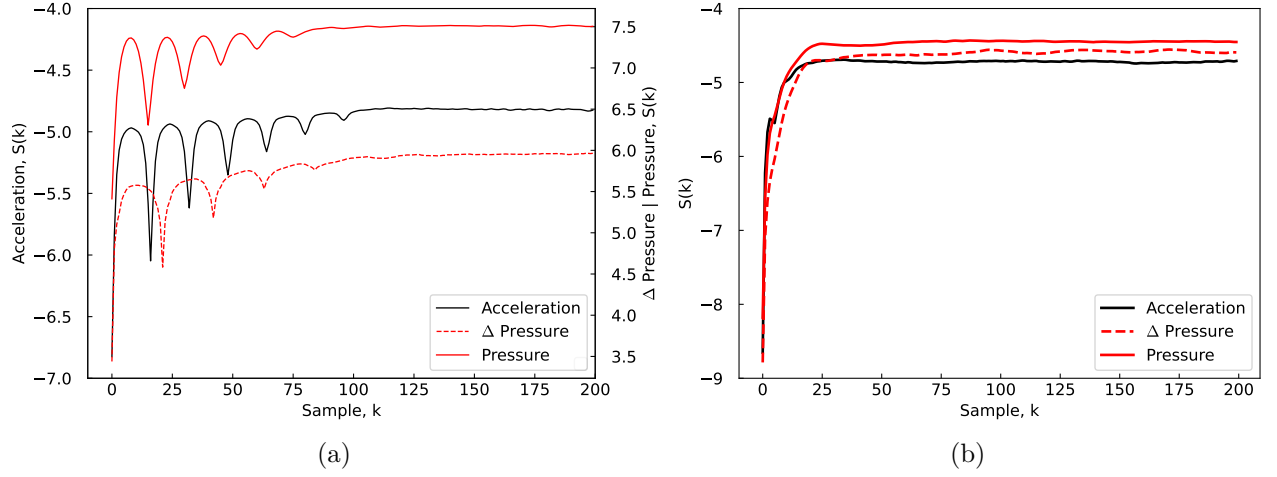


Figure 6.31 – Curve $S(k)$ versus k from the pressure gradient, pressure, and acceleration signals for the reconstructed state space using the consecutive delay method (a) and using the three main singular values of the decomposition of singular value (b), for experimental point P13.

Figure 6.32 presents the largest Lyapunov exponent values obtained from experimental data points in slug flow patterns and for all measured quantities (acceleration, pressure, and pressure gradient). Remarkably, these values, in all cases, are positive, indicating chaotic behaviour in which trajectories exponentially diverge. Furthermore, it is observed that the Lyapunov exponent values are consistent among the three different measurements, suggesting that chaotic behaviour is not specific to a particular sensor or measurement variable but rather an intrinsic characteristic of the entire system. The vast majority of data points fall within a variation range that lies within $\pm 15\%$ limits concerning the polynomial fit, Table 8 contains all the values of the Largest Lyapunov exponent.

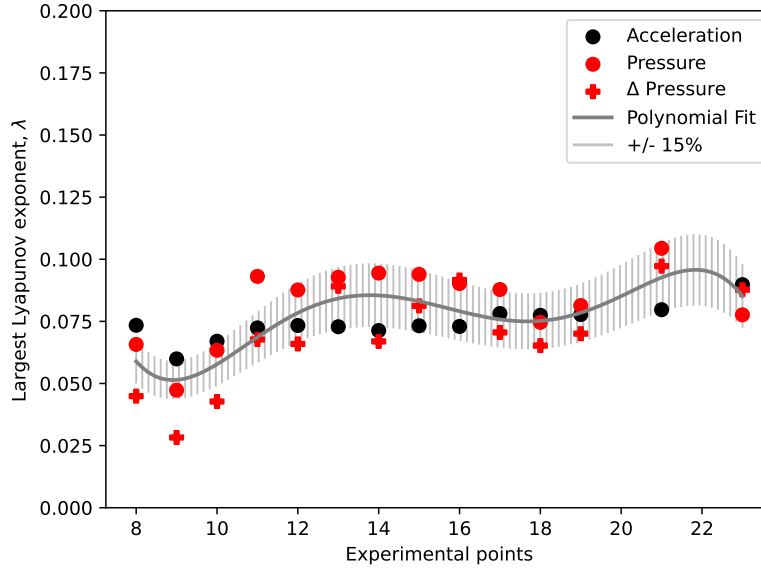


Figura 6.32 – Largest Lyapunov exponent values for pressure (P_{re10}), pressure gradient $\Delta(P_{re3} - P_{re8})$, and acceleration (Ac_1) for all points of the slug pattern in the test matrix.

Tabela 8 – Largest Lyapunov exponent (λ) estimates from pressure (P_{re10}), pressure gradient $\Delta(P_{re3} - P_{re8})$, and acceleration (Ac_1) signals for all points of the slug pattern in the test matrix.

Point	λ - Acceleration	λ - Δ Pressure	λ - Pressure
8	0.073	0.045	0.066
9	0.060	0.028	0.047
10	0.067	0.043	0.063
11	0.072	0.068	0.093
12	0.073	0.066	0.088
13	0.073	0.089	0.093
14	0.071	0.067	0.094
15	0.073	0.081	0.094
16	0.073	0.092	0.090
17	0.078	0.071	0.088
18	0.077	0.065	0.075
19	0.078	0.070	0.081
21	0.080	0.097	0.104
22	0.090	0.088	0.078

The largest Lyapunov exponent is a metric used to characterise deterministic chaos in dynamical systems. It is an invariant measure, meaning it is independent of the specific system, measurement process, and state space reconstruction. Therefore, it is interesting to note that the values in Table 8 exhibit low variance for the same measurement point, even though they correspond to experimental data obtained from different sensors.

6.4.5 Coefficient of correlation dimension

The coefficient of correlation dimension (c) can be employed to assess the complexity of dynamic systems exhibiting varying degrees of complexity, such as distinguishing between low-dimensional and high-dimensional systems. The [Grassberger e Procaccia \(1983\)](#) method was used to calculate the c , as discussed in Section 4.4.4. Figure 6.33 illustrates the relationship between the Correlation Dimension (CD) and the radius (ϵ) calculated from Equation 4.43 of the pressure gradient $\Delta(P_{re1} - P_{re6})$, pressure (P_{re10}) and acceleration (Ac_1) signals, at experimental point P13. The correlation coefficient is determined as the slope of the initial portion of the logarithmic curve of CD *versus* ϵ . Remarkably, the values of d_e vary in the range from 1 to 12, and the state space was reconstructed based on the principal singular values of the decomposition of singular value, and consequently, the adopted time delay was unitary.

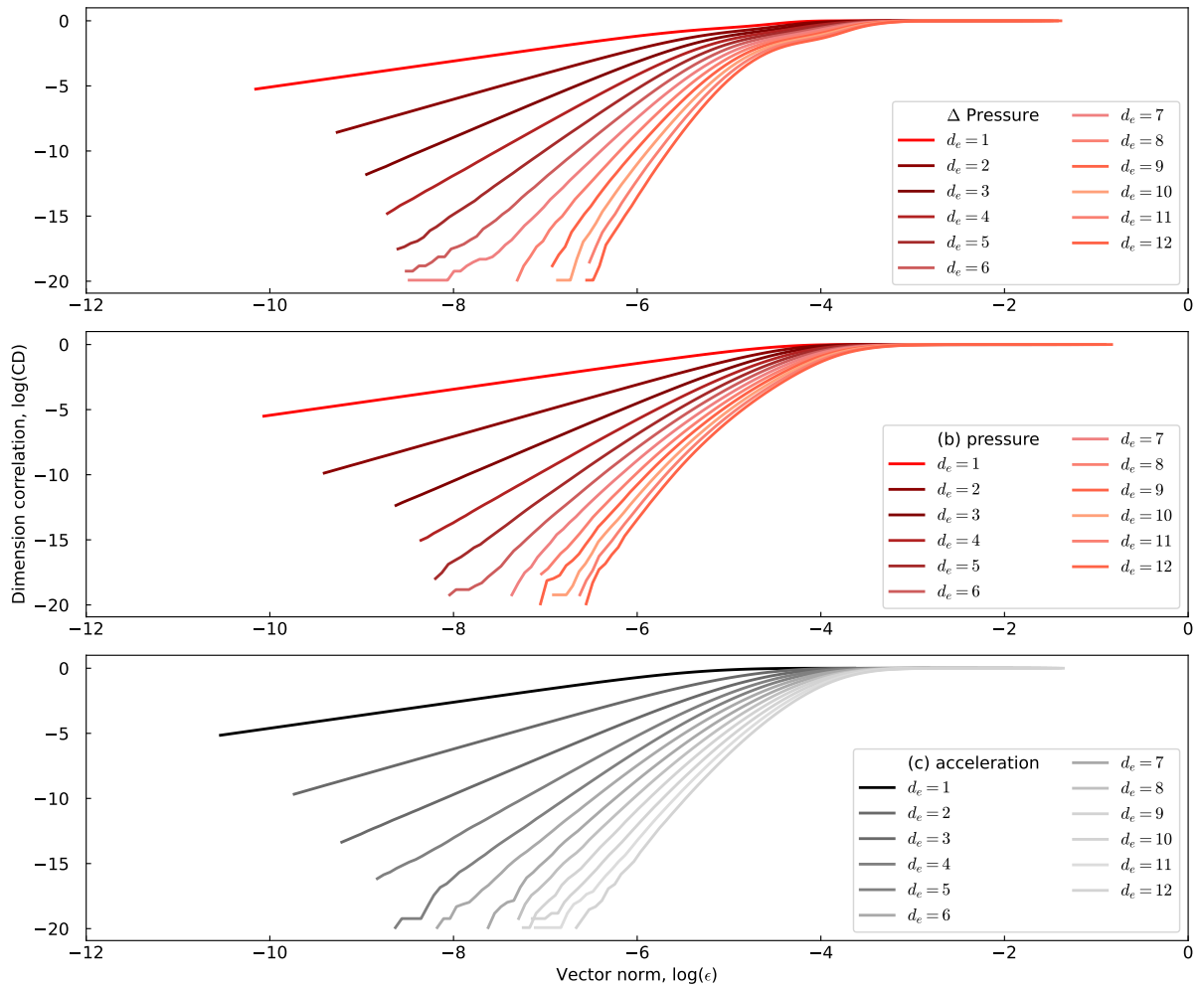


Figura 6.33 – Logarithmic curve of the Correlation Dimension (CD) as a function of the radius (ϵ) for pressure gradient (a), pressure (b), acceleration signals (c) for experimental point P13.

Figure 6.34 presents the relationship between the value of c and d_e for all signals. Note that the value of c increases with the increasing embedding dimension, and the values

for all signals are relatively close, following the same trend. A quadratic fit was applied to analyse the data trend, demonstrating that the majority of data points fall within a 15% range of variation. Table 9 presents the values of c for all signals, for d_e equal to 6 as obtained by the Broomhead method, as discussed in the previous section. It's worth noting that the values are close for the same experimental point. This may indicate that the measurements obtained by these sensors are correlated and vary similarly in response to the system's conditions. In practical terms, this correlation could suggest that changes in one variable (such as pressure, acceleration, or void fraction) are related to changes in the other variables. This can be useful for understanding the system's behavior, identifying patterns or anomalies, and making informed decisions based on these measurements.

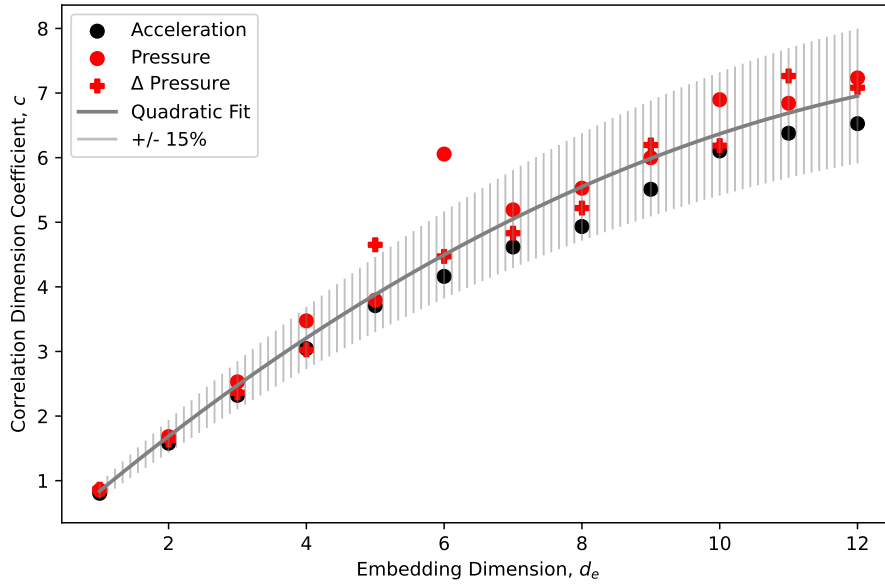


Figura 6.34 – The coefficient of correlation dimension (c) as a function of the estimated embedding dimension (d_e) for the acceleration (Ac_1), pressure (P_{re10}), and pressure gradient $\Delta(P_{re1} - P_{re6})$ sensors for experimental point P13.

Tabela 9 – Coefficient of correlation dimension (c) for the slug point of the test matrix from acceleration (Ac_1), pressure (P_{re10}) and pressure gradient $\Delta(P_{re1} - P_{re6})$ for $d_e = 6$.

Point	c - Acceleration	c - Δ Pressure	c - Pressure
8	4.089	3.876	2.635
9	4.123	3.480	2.669
10	4.402	3.872	3.009
11	3.692	5.247	5.316
12	5.516	4.705	5.078
13	4.379	5.648	6.051
14	4.162	4.471	6.056
15	4.150	3.626	5.455
16	4.126	4.778	5.608
17	4.372	4.90	5.700
18	5.108	4.541	5.776
19	4.762	5.738	4.568
21	4.159	5.266	5.873
22	4.792	5.846	5.713

6.5 Flow pattern classification from acceleration measurements

In this section, a flow pattern classification approach is proposed based on the time domain signals obtained from a single accelerometer. To obtain the results, the signals were demodulated, as shown in the previous section. Moreover, the signals were down sampled at up to 50 Hz, maintaining the duration of the time series at 600 s. For that, a second-order Butterworth digital filter was applied to avoid aliasing effects.

6.5.1 Root mean square - acceleration signals

The Root Mean Square (RMS) value was calculated from the demodulated acceleration signal, and the result is shown in Figure 6.35. The results are grouped by experimental point, and for each experimental point, there are five bars representing the five accelerometers present in the duct, sequentially from Ac_1 to Ac_5 . The colours indicate the flow pattern: red represents smooth stratified (SS) pattern, green represents wavy stratified (SW) pattern, blue represents slug (SL) pattern, and yellow represents dispersed bubble (DB) pattern. As expected, the stratified and dispersed bubble patterns have a much lower RMS value when compared to slugs because these patterns involve a low energy level in the system. Table 10 presents the RMS values for each experimental point for the Ac_1 acceleration signal.

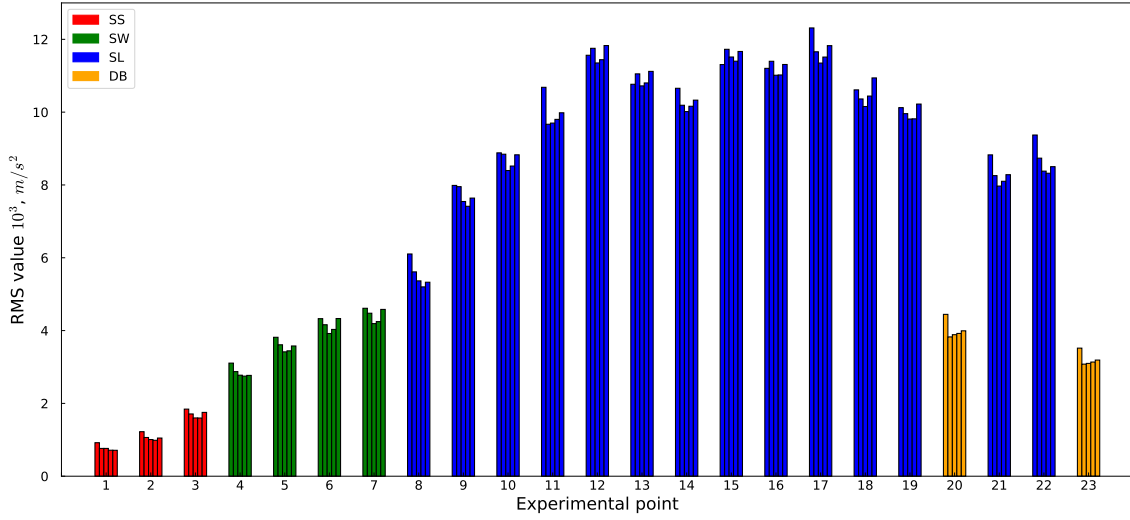


Figure 6.35 – Root Mean Square (RMS) values calculated from demodulated vibration measurements for all 23 experimental points in the test matrix. Each bar represents, sequentially, an acceleration sensor: Ac_1 , Ac_2 , Ac_3 , Ac_4 , and Ac_5 , and each color represents a flow pattern: red for smooth stratified (SS), green for wavy stratified (SW), blue for slug (SL), and yellow for dispersed bubbles (DB).

Therefore, it can be observed that using RMS values only to classify patterns may present some issues, the main one being the failure to consider characteristics of the dynamic system or relationships between time series at different time points. Additionally, RMS values calculate the mean of the squares of the values, which can result in the loss of relevant information about the distribution and shape of the time series, compromising the ability to identify subtle or complex patterns in the time series. Nevertheless, good coherence is observed among different pressure sensors along the line, suggesting good repeatability.

6.5.2 Hurst exponent - acceleration signals

Long-term dependence, or its trend, is a characteristic of a time series that is often quantified by the Hurst exponent. When a time series has a Hurst exponent greater than 0.5, it means it has long-term memory, also known as persistence, which means its past increments influence future ones, and the process tends to maintain the sign of the increments. Therefore, even though it is a probabilistic process, this observed time series may oscillate around a certain value, the process still has long-term memory, which is a global feature. On the other hand, if the exponent has values less than 0.5, the process is anti-persistent and has short-term memory. This means that the values observed in the time series generally change from relatively high to relatively low values. The Hurst exponent is also related to the autocorrelation of the time series, i.e., the rate of its decrease; a persistent process is sometimes also called correlated, and an anti-persistent one is called anti-correlated.

The rescaling technique, R/S (Range over Standard Deviation), was used to calculate the Hurst exponent (h). In summary, this technique is based on calculating the relationship between a period and the standard deviation of the time series at different interval sizes or subdivisions (k). Firstly, the time series is divided into different sizes, and for each interval, the R/S value is calculated, where R is the difference between the maximum and minimum values within the interval, and S is the standard deviation of the values within the interval. Then, the Hurst exponent is given by the slope of this log-log curve of k against R/S . Figure 6.36 provides the result of the rescaled range of the series (R/S) and the subdivisions k for all points in the test matrix using the P_{rel10} sensor. Note that in the linear fit (solid red line) of the points, the slope corresponds to the Hurst exponent. The total number of subdivisions used was 40, and the minimum window size adopted was 5 seconds (250 segments), while the maximum was 60 seconds (3000 segments), varying linearly within this range. The key point for analysis is the choice of the maximum and minimum values for the subdivision interval of the time series, as well as the total number of subdivisions, as it needs to capture both the smallest and largest scales of system dynamics. Therefore, observe that the choice of the window was appropriate, as the linear fit showed good agreement for all points in the experimental matrix.

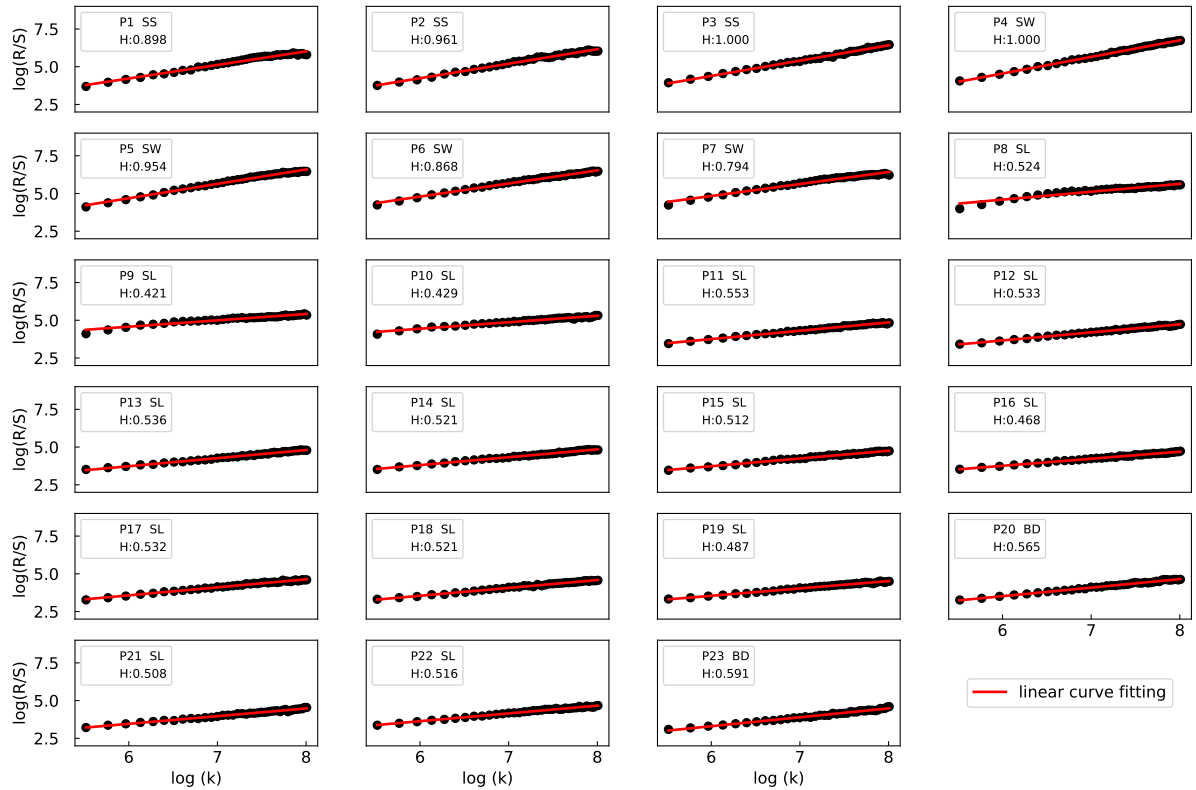


Figure 6.36 – Logarithmic curve of the ratio between the rescaled range of the series (R/S) and the subdivisions (k) for all 23 experimental points for the demodulated acceleration signal from sensor Ac_1 ; the solid red line represents the linear fit of the points, with its slope being the value of the Hurst exponent.

Figure 6.37 presents the Hurst coefficient values for all experimental points in the

test matrix, calculated using R/S techniques. It is possible to observe that the values for the stratified points correspond to values greater than $0.5 < h < 1$, indicating persistence in the series. This means that the system in the stratified pattern does not exhibit significant oscillations, maintaining its nearly constant excitation characteristic. The same result was obtained for the pressure sensors, shown later in the Chapter. In contrast, for the intermittent and dispersed bubble patterns, the Hurst exponent values are very close to 0.5, indicating non-persistent behaviour similar to a random Brownian motion. However, it is noteworthy that there was a clear separation between the stratified patterns and the intermittent and dispersed bubble patterns. Table 10 summarises the exponent values for each experimental point from the Ac_1 acceleration signal.

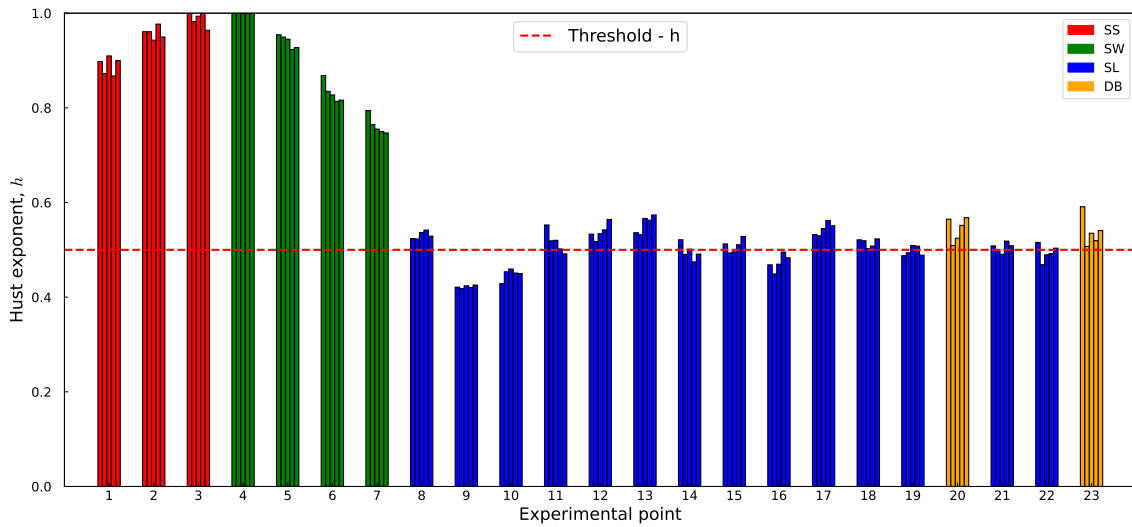
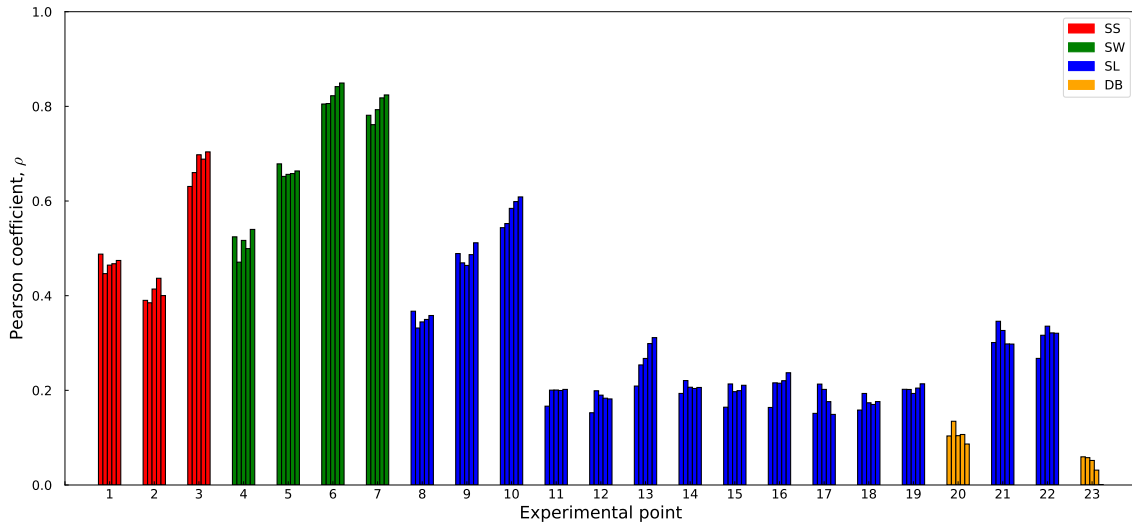


Figure 6.37 – Hurst exponent (h) calculated for demodulated acceleration signals for all 23 experimental points in the test matrix. Each bar represents a sequentially numbered acceleration sensor: Ac_1 , Ac_2 , Ac_3 , Ac_4 , and Ac_5 , and each color represents a flow pattern: red for smooth stratified (SS), green for wavy stratified (SW), blue for slug (SL), and yellow for dispersed bubbles (DB).

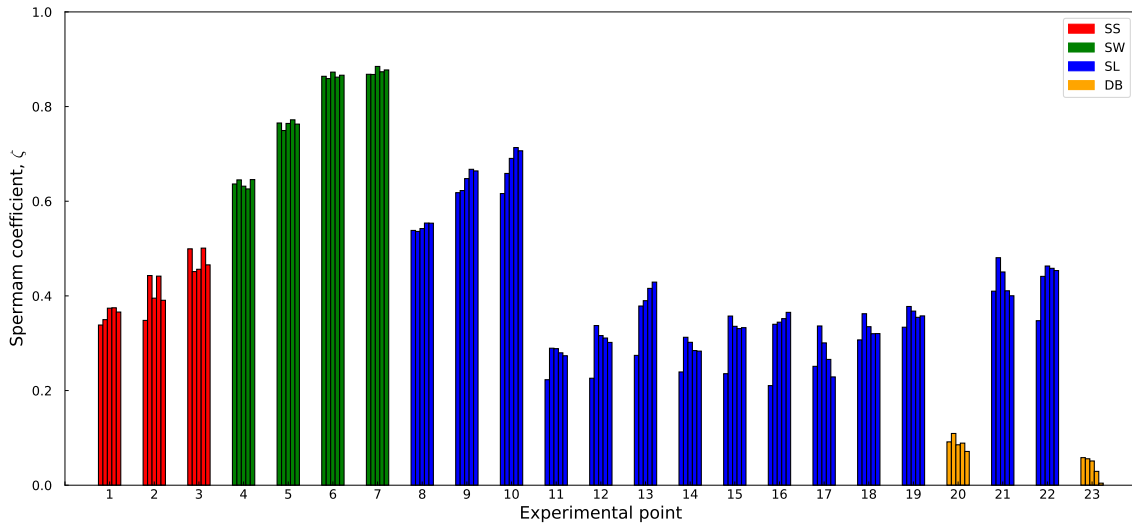
6.5.3 Pearson and Spearman coefficient - acceleration signals

The Pearson coefficient (ρ) is a parametric measure that assesses the linear relationship between two variables, while the Spearman coefficient (ζ) is a non-parametric measure that evaluates the monotonic association between two variables (SHIN; HAMMOND, 2008). Both coefficients range from -1 to 1, where -1 indicates a perfect negative correlation, 1 indicates a perfect positive correlation, and 0 indicates no correlation. Figure 6.38 presents the result of calculating the ρ (a) and the ζ (b) for all experimental points in the test matrix. As in previous results, each vertical bar represents a sensor, totalling five, and the colours represent the flow patterns, as identified in Table 3. In this case, the two variables are the demodulated acceleration signals at two typical structural frequencies, namely the first and second cut-on frequencies. Note that in all cases, the values are positive, indicating positive correlations, meaning that both variables move

in the same direction. However, for the dispersed bubbles pattern, the coefficient is very close to zero, indicating no association between the two variables. Some studies have used this technique to distinguish dispersed bubble patterns from other patterns (CARVALHO; FIGUEIREDO; SERPA, 2019; SESTITO et al., 2023), but they used lower filtering frequencies that were unrelated to structural responses. Here, the two typical demodulation frequencies of the structure are adopted, the cut-on frequencies. However, it can be observed that the patterns were consistently distinguished, with SS and SW patterns having higher ρ values and DB pattern values close to zero. Table 10 presents the coefficient values for each experimental point from the acceleration signal Ac_1 .



(a)



(b)

Figura 6.38 – Pearson coefficient (ρ) (a) and Spearman coefficient (ζ) (b) calculated from demodulated acceleration measurements for all 23 experimental points in the test matrix. Each bar represents a sequentially numbered acceleration sensor: Ac_1 , Ac_2 , Ac_3 , Ac_4 , and Ac_5 , and each color represents a flow pattern: red for smooth stratified (SS), green for wavy stratified (SW), blue for slug (SL), and yellow for dispersed bubbles (DB).

6.5.4 Three-dimensional pattern map - acceleration signals

The parameters extracted from the demodulated vibration signals are the Hurst exponent, Pearson coefficient, and Spearman coefficient. As the test matrix comprises 23 experimental points, and the test setup has 5 acceleration sensors, it was possible to test 115 points using these techniques. Figure 6.39 presents the obtained results, where, once again, colours indicate the flow pattern observed in the video footage, and the symbol represents each sensor on the test setup. Several clusters of points with the same flow patterns can be observed. The smooth and wavy stratified patterns are very close to each other and distant from the slug pattern cluster. Additionally, the dispersed bubbles pattern is also well-grouped and separated from other patterns. However, points P8, P9, and P10 formed a new cluster located between the stratified and slug patterns; these points are considered transition points, as they are slugs with extremely long bubbles, as indicated in Table 13. All parameter values for each experimental point are summarised in Table 10 using accelerometer sensor Ac_1 .

Tabela 10 – Dimensionless parameters of Pearson coefficient (ρ), Spearman coefficient (ζ), and Hurst exponent (h) for the 23 experimental points of acceleration sensor Ac_1 .

P	J_{sl} [m/s]	J_{sg} [m/s]	RMS, $\times 10^3$ m/s²	h	ρ	ζ	Pattern
1	0.258	0.106	0.920	0.898	0.488	0.338	SS
2	0.259	0.203	1.223	0.961	0.390	0.348	SS
3	0.258	0.481	1.843	1.000	0.631	0.499	SS
4	0.258	0.899	3.107	1.000	0.524	0.636	SW
5	0.258	1.145	3.816	0.954	0.679	0.765	SW
6	0.258	1.822	4.327	0.868	0.805	0.864	SW
7	0.259	2.036	4.613	0.794	0.781	0.868	SW
8	0.580	0.504	6.104	0.524	0.367	0.538	SL
9	0.579	0.977	7.987	0.421	0.489	0.618	SL
10	0.578	1.429	8.882	0.429	0.544	0.616	SL
11	1.191	0.382	10.682	0.553	0.167	0.223	SL
12	1.315	0.845	11.562	0.533	0.153	0.226	SL
13	1.353	1.400	10.766	0.536	0.209	0.274	SL
14	1.300	0.493	10.655	0.521	0.193	0.239	SL
15	1.242	1.001	11.306	0.512	0.164	0.236	SL
16	1.210	1.218	11.203	0.468	0.164	0.210	SL
17	2.150	0.532	12.313	0.532	0.151	0.251	SL
18	2.044	1.017	10.612	0.521	0.158	0.307	SL
19	2.017	1.182	10.124	0.487	0.202	0.334	SL
20	3.153	0.095	4.445	0.565	0.104	0.092	DB
21	3.042	0.421	8.826	0.508	0.301	0.410	SL
22	2.927	0.854	9.372	0.516	0.267	0.348	SL
23	3.328	0.078	3.518	0.591	0.060	0.058	DB

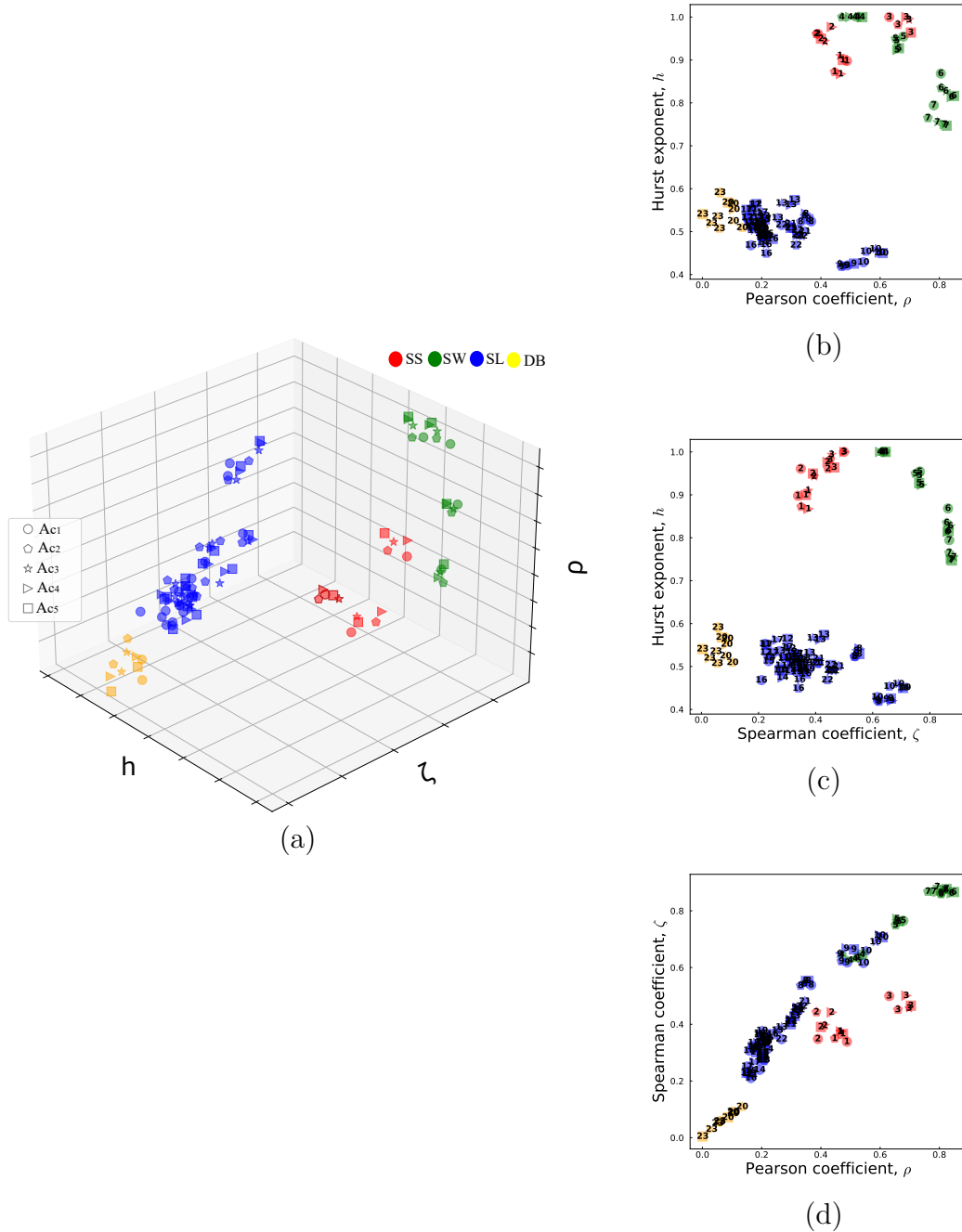


Figura 6.39 – Three-dimensional flow pattern map: Pearson coefficient (ρ), Spearman coefficient (ζ), and Hurst exponent (h) calculated from demodulated acceleration signals for all 23 experimental points for all sensors. Please note that each symbol represents a pressure sensor from Ac_1 to Ac_5 , and each color represents a flow pattern: red for smooth stratified (SS), green for wavy stratified (SW), blue for slug (SL), and yellow for dispersed bubbles (DB).

6.5.5 Application to field data from a refinery

Vibration measurements were conducted on pipelines at a Petrobras Oil&Gas refinery. Various pipelines and measurement points within the refinery were utilised, featuring diameters of 12, 16, and 20 inches, all equipped with thermal insulation. The

tests were divided into two days: the first was conducted on day 1 under normal operational conditions, and the second on day 2, deliberately inducing anomalous plant operation. It is important to emphasise that confirming the flow pattern was not possible, as the industrial plant lacks appropriate equipment or a viewing section. The only available information comes from the engineers responsible, who suggest that the flow pattern varies between stratified, dispersed bubbles and slugs.

Exploring the fluid-structure coupling phenomenon, shell models were analytically computed and then demodulated. Similarly to the previous section, Pearson, Spearman, and Hurst exponent thresholds were calculated from the demodulated signals, and the results are displayed in Figure 6.40. Note that the results are presented alongside the three-dimensional map shown in Figure 6.39, indicating with indicated with coloured circles (PAPE). A hypothesis is made that the flow patterns in the refinery measurements can be classified based on the map generated in the experimental setup. In this regard, it should be noted that measurements M18, M16, and M17, from point P3 on day 2, are classified in the dispersed bubble pattern. The other points are in a region of the map in the stratified zone and transitioning to the piston-like pattern.

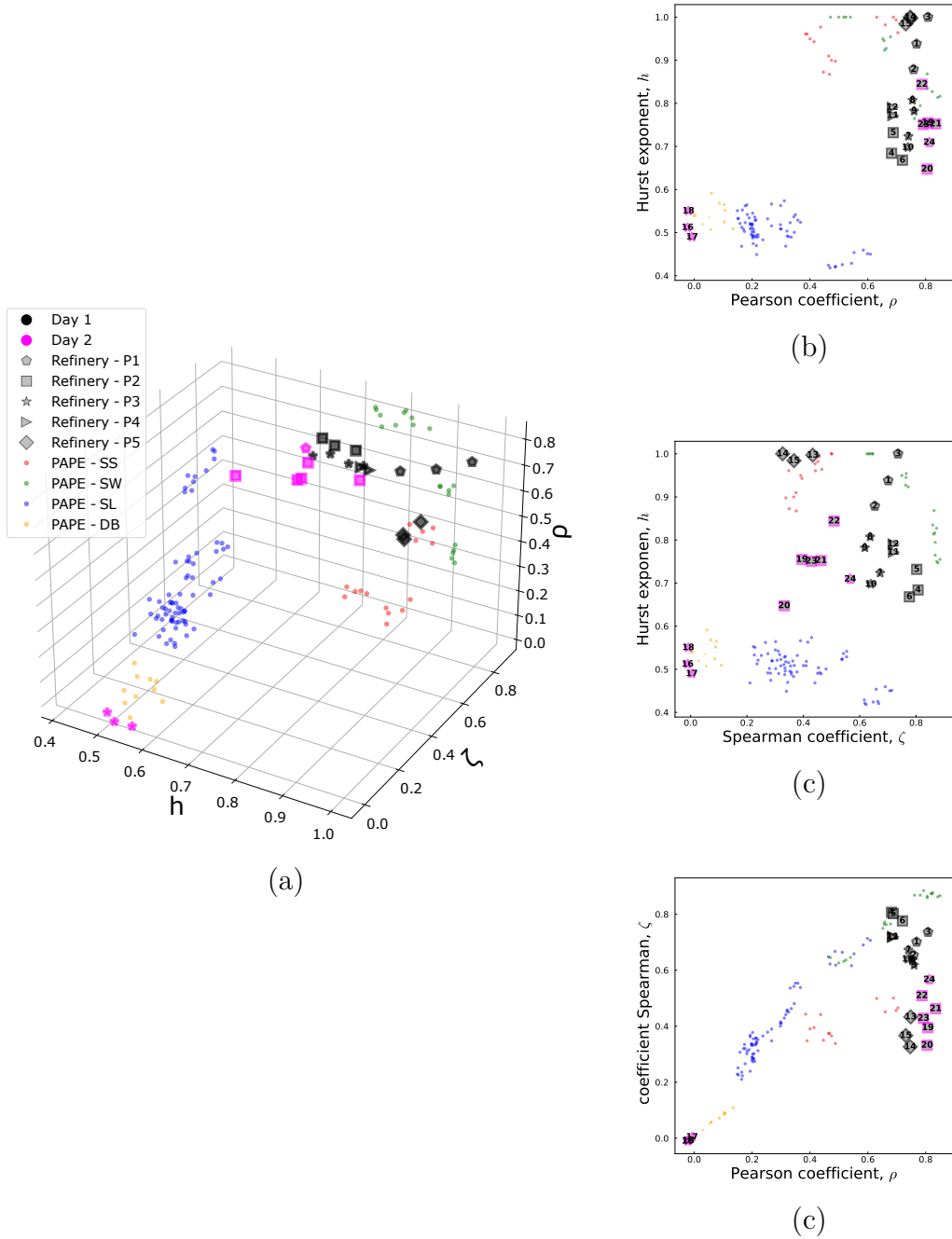


Figura 6.40 – Three-dimensional flow pattern map: Pearson coefficient (ρ), Spearman coefficient (ζ), and Hurst exponent (h) calculated from the demodulated vibration signals of the refinery.

This field application of the laboratory-developed methodology at the refinery encountered three key challenges compared to laboratory bench testing. Firstly, the refinery deals with multiphase fluids, unlike the laboratory's two-phase setup. Secondly, Laser Vibrometers (LDV) replaced accelerometers for velocity measurements, resulting in linear frequency response decay and high-frequency noise issues due to field conditions. Lastly, the refinery's pipes featured thick ceramic insulation layers, potentially attenuating high-frequency responses.

Despite these challenges, the vibration data analysis enabled flow pattern identification at each measurement point under two refinery operational conditions. Validation of this methodology for field scenarios is recommended through comparison with commercially mature meters, especially in Exploration & Production (E&P) contexts.

6.6 Flow pattern classification from pressure measurements

In all the analyses in this section, the piezoelectric pressure signals are downsampled at a rate of 50 Hz, while maintaining the duration of the series at 600 seconds. In this downsampling a second-order Butterworth digital filter is applied to prevent aliasing effects. The objective of this section is to investigate the identification of flow patterns using techniques to calculate the Hurst exponent, correlation dimension coefficient, and Lyapunov exponent. For this purpose, all experimental data points from all sensors and pressure measurements are utilised, resulting in the testing of 230 data series.

6.6.1 Root mean square - pressure signals

Figure 6.41 shows RMS values calculated from the pressure time series as a function of the experimental points. All pressure sensors were analysed, from P_{re1} to P_{re10} , and all experimental points, from P1 to P23. The colours represent different flow patterns: red indicates smooth stratified flow (SS), green indicates wavy stratified flow (SW), blue indicates the slug flow pattern (SL), and yellow indicates the dispersed bubble flow pattern (DB). Each vertical bar represents a pressure sensor, arranged sequentially from P_{re1} to P_{re10} . It is possible to observe that the RMS value varies depending on the flow pattern. Note that the experimental points P17, P18, P19, P20, P21, and P22 are of the slug flow pattern and have the highest RMS values because they are points with the highest mixing velocity in the test matrix. SS flow patterns have lower RMS values when compared to other patterns, and SW and DB patterns have similar levels of RMS values. However, the points from P8 to P16, despite being part of the slug pattern, have RMS values very close to stratified patterns.

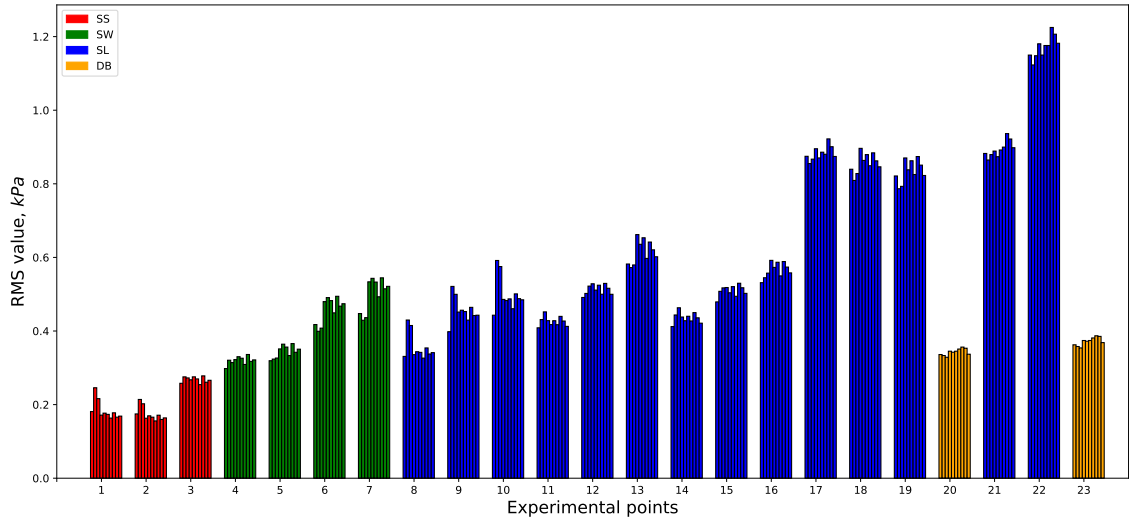


Figura 6.41 – Root Mean Square (RMS) value calculated from the pressure time series for all 23 experimental points in the test matrix and sensors. The values are grouped into sets of ten vertical bars, with each bar representing a sensor, arranged sequentially from P_{re1} to P_{re10} . Each color represents a flow pattern: red for smooth stratified (SS), green for wavy stratified (SW), blue for slug (SL), and yellow for dispersed bubbles (DB).

Therefore, it can be observed that the RMS values of the demodulated acceleration signals have a better ability to segregate flow patterns when compared to pressure signals, as shown in Figure 6.35. However, as discussed earlier, the use of RMS values for pattern classification may overlook key dynamic system characteristics and inter-time point relationships, potentially leading to a loss of information about time series distribution and pattern complexity. Despite this limitation, there is notable coherence among pressure sensors along the line, indicating good repeatability.

6.6.2 Hurst exponent - pressure signals

In this section, the results obtained with the Hurst exponent (h) are presented and discussed for the case of the pressure sensor. Figure 6.42 shows the logarithmic curve of R/S values and the number of subdivisions k , obtained using the procedure shown in Section 4.2.7. The red curve represents the linear fit of the points, where the slope corresponds to the h value. It used a total of 40 subdivisions, with the largest window being 5 seconds and the smallest 60 seconds, the same values adopted for the acceleration signal.

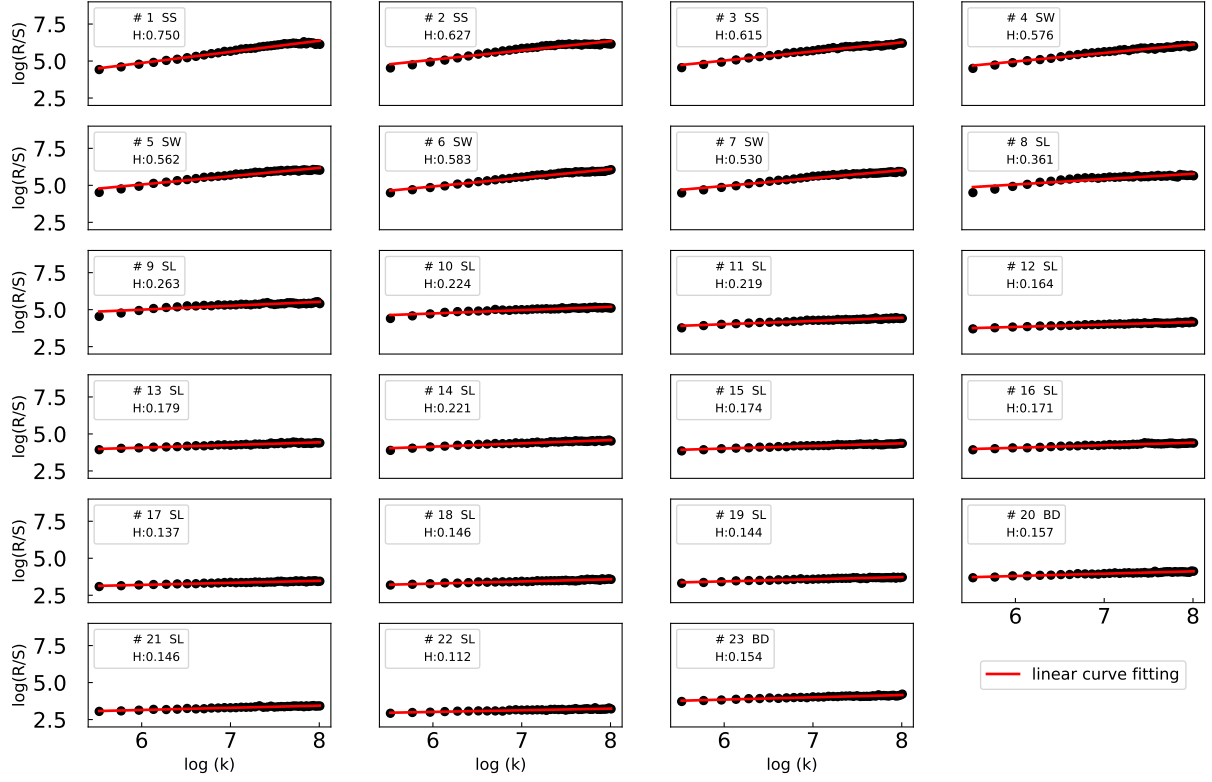


Figure 6.42 – Logarithmic curve of the ratio between the rescaled range of the series (R/S) and the subdivisions (k) for all 23 experimental points for the demodulated acceleration signal from sensor Pre_{10} . The solid red line represents the linear fit of the points, with the slope corresponding to the value of the Hurst exponent.

Figure 6.43 shows the value of h for each point in the test matrix for all pressure sensors. The results are grouped by experimental point, and each vertical bar represents a pressure sensor distributed sequentially from P_{re1} to P_{re10} . Note that the values for stratified patterns are between $1 > h > 0.5$, indicating a persistent series. Regarding intermittent patterns, the values of h range between $0 < h < 0.5$, corresponding to an anti-persistent series. This behavior is associated with the oscillatory nature of this pattern, alternating between the liquid slug and the Taylor bubble, resulting in exponent values below 0.2 for most points in the slug pattern. However, observe that for experimental points P8 and P9, the values of h are slightly higher than the other points in the slug pattern but still below 0.5. In these points, the Taylor bubble is larger, indicating a higher incidence of the stratified region. The values of h can be seen in Table 13 for pressure sensor P_{re10} .

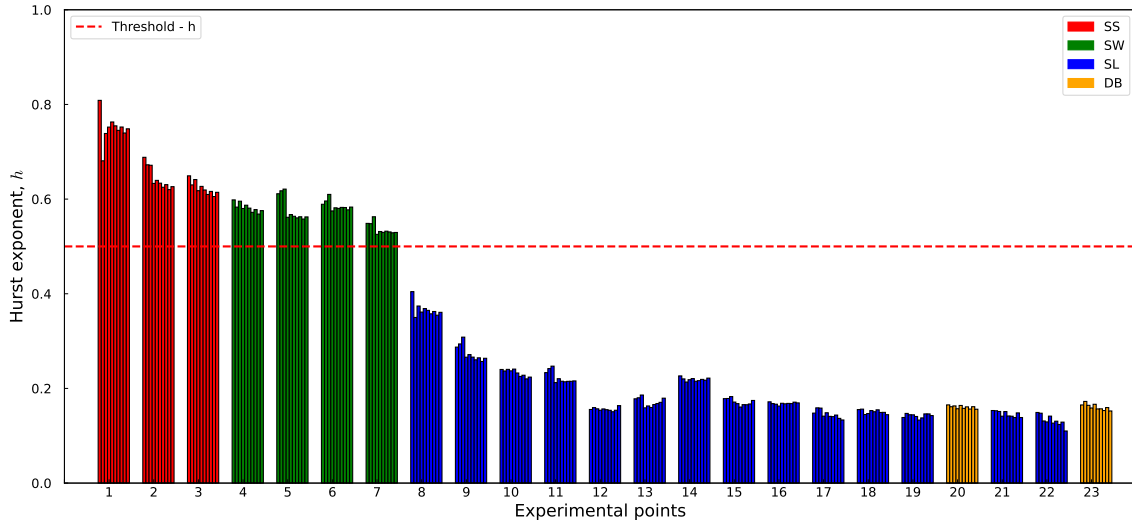


Figura 6.43 – Value of the Hurst exponent (h) calculated from the pressure time series for all 23 experimental points in the test matrix and sensors. The values are grouped into sets of ten vertical bars, with each bar representing a sensor, distributed sequentially from P_{re1} to P_{re10} . Each color represents a flow pattern: red for the smooth stratified (SS), green for the wavy stratified (SW), blue for the slug (SL), and yellow for the dispersed bubbles (DB).

For acceleration signals, the h values in the stratified patterns exceed 0.5, as shown in Figure 6.37, consistent with the results obtained from the pressure signals. However, in the intermittent pattern, the acceleration signals showed values close to the 0.5 threshold, whereas the pressure signals displayed h values below 0.5. This highlights a significant difference in the temporal signature obtained with pressure and acceleration sensors, especially in the case of the slug pattern, caused by transients in the acceleration signals and the excitation mechanism of the duct due to the presence of dispersed bubbles in both the liquid slug and the dispersed bubble pattern.

6.6.3 Largest Lyapunov exponent - pressure signals

The calculation of the largest Lyapunov exponent (λ) was performed using the state space reconstructed from singular values, employing the three principal eigenvectors obtained from the decomposition. Note that in Figure 6.44, all patterns, including stratified, intermittent, and dispersed bubble patterns, exhibited positive values for the Lyapunov exponent. This indicates that all flow patterns display chaotic behaviour, in which particle trajectories exponentially diverge from each other over time. The positive values of the largest Lyapunov exponent suggests the presence of turbulence and complexity in all three types of flow patterns, which is characteristic of chaotic systems. The stratified flow points exhibit lower λ values compared to the intermittent points due to the intrinsic characteristics of the two flow pattern types. The Lyapunov exponent measures the rate of divergence of trajectories in the state space, where positive values indicate chaotic behaviour and rapid trajectory divergence. Conversely, SL and DB flow patterns are characterised

by greater instability and turbulence. Particle trajectories can quickly diverge from each other, resulting in a higher rate of divergence and, consequently, larger λ values.

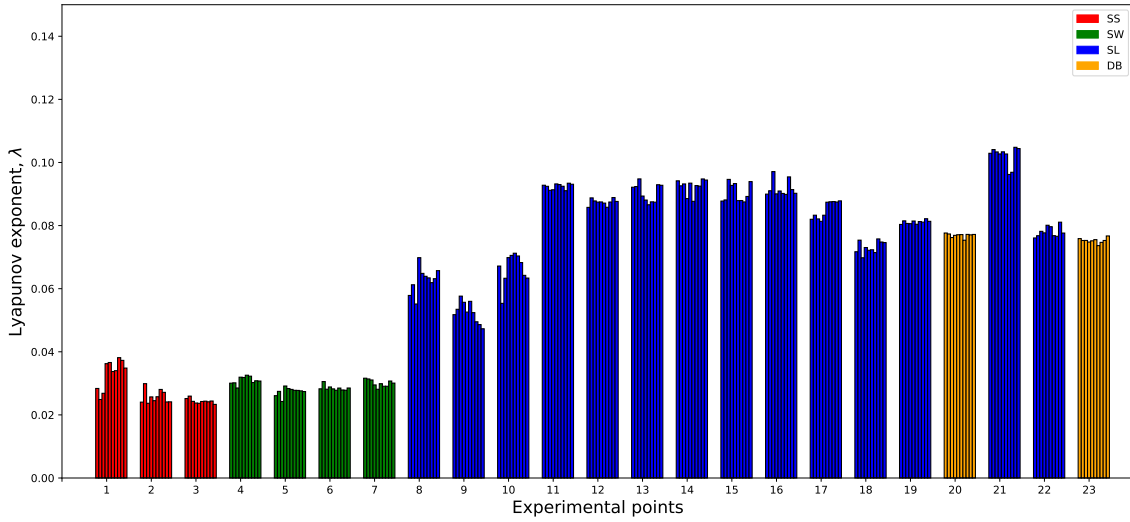


Figure 6.44 – Largest Lyapunov exponent (λ) calculated from the pressure time series for all 23 experimental points in the test matrix and sensors. The values are grouped into sets of ten vertical bars, with each bar representing a sensor, distributed sequentially from P_{re1} to P_{re10} . Each color represents a flow pattern: red for the smooth stratified (SS), green for the wavy stratified (SW), blue for the slug (SL), and yellow for the dispersed bubbles (DB).

Therefore, even though the Lyapunov exponent values may differ between stratified and intermittent patterns, both share the fundamental characteristic of exhibiting chaotic behaviour in which trajectories diverge exponentially. The difference in Lyapunov exponent values reflects the rate of trajectory divergence, which may be higher in intermittent patterns due to the increased turbulence and instability associated with these patterns.

6.6.4 Coefficient of correlation dimension - pressure signals

As previously mentioned, the coefficient of correlation dimension (c) is a tool to quantify self-similarity when it exists in a time series. When applied to pressure signals from a two-phase flow, the c represents a measure of the complexity of the reconstructed attractor from the time series in question. Therefore, large c values of the time series of a given signal indicate more complex shapes in its attractor. In [Neves et al. \(2023\)](#), c is calculated through the reconstruction of the attractor using the method of delays, and the time lag is determined through the autocorrelation function. As a result, the authors were able to establish a significant distinction amongst the flow patterns. The patterns referred to as SS and SW exhibited values below 4, while the intermittent patterns displayed values exceeding 4. This enabled a clear demarcation and differentiation among the various investigated flow patterns. In this work, the correlation dimension was calculated using the attractor reconstructed from the principal components resulting from the singular

value decomposition of the pressure signals. Figure 6.45 illustrates c for all sensors and all points in the test matrix. Similar to the referenced work, a threshold value of 4 is observed, distinguishing between stratified patterns and dispersed bubbles. However, in contrast to the previously mentioned study, the values for the SS patterns were found to be greater than those for the SW patterns. This variation is attributed to the different methods of state space reconstruction.

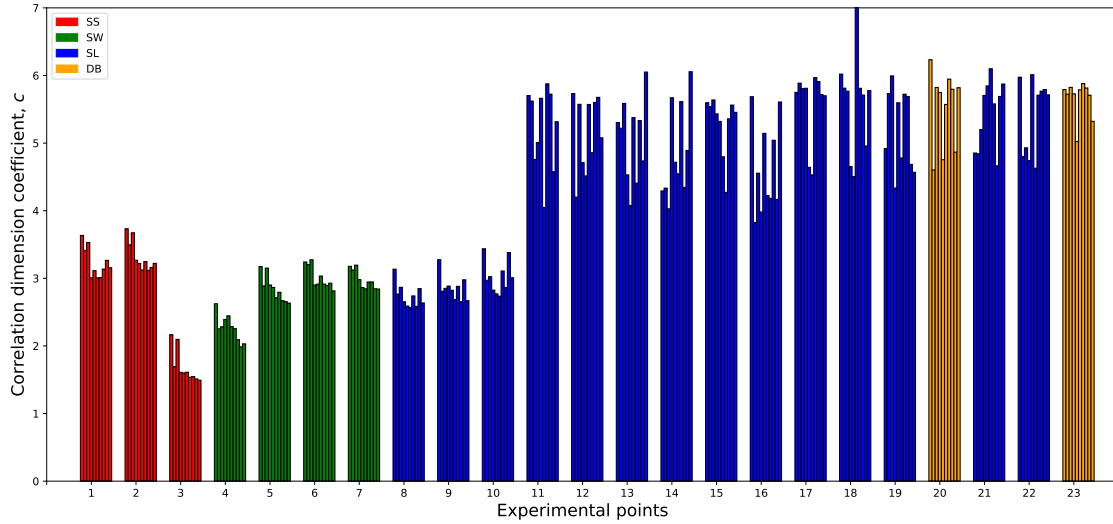


Figure 6.45 – Coefficient of correlation dimension (c) calculated from the pressure signals of all 23 experimental points for all sensors. The different symbols indicate the sensors from P_{re1} to P_{re10} . The values are grouped into sets of ten vertical bars, with each bar representing a sensor, distributed sequentially from P_{re1} to P_{re10} . Each color represents a flow pattern: red for the smooth stratified (SS), green for the wavy stratified (SW), blue for the slug (SL), and yellow for the dispersed bubbles (DB).

6.6.5 Three-dimensional pattern map - pressure signals

The presented dimensionless parameters, the Hust exponent (h), the Largest Lyapunov exponent (λ), and the coefficient of correlation dimension (c) are calculated from the measurements of all of the sensors and at all of the test points of the test matrix, and these are given in Figure 6.46, a total of 230 time series were tested. Notice that a clustering of the points with the same flow patterns is clearly formed and the points from different sensors from the same experimental run are consistently close to each other. Most remarkably, the stratified-smooth and stratified-wavy patterns are very close together and well-distanced from the slug pattern cluster. While the points in the low-frequency SL patterns P8, P9, and P10 are in a region between the two corresponding clusters. In addition, the cluster of dispersed bubbles is closer to the cluster of slug pattern, which is expected since the latter also presents dispersed bubbles in the liquid piston. This indicates that clustering has some physical meaning due to the choice of dimensionless parameters. All dimensionless parameters for each experimental point are summarised in Table 11 using P_{re10} pressure sensor.

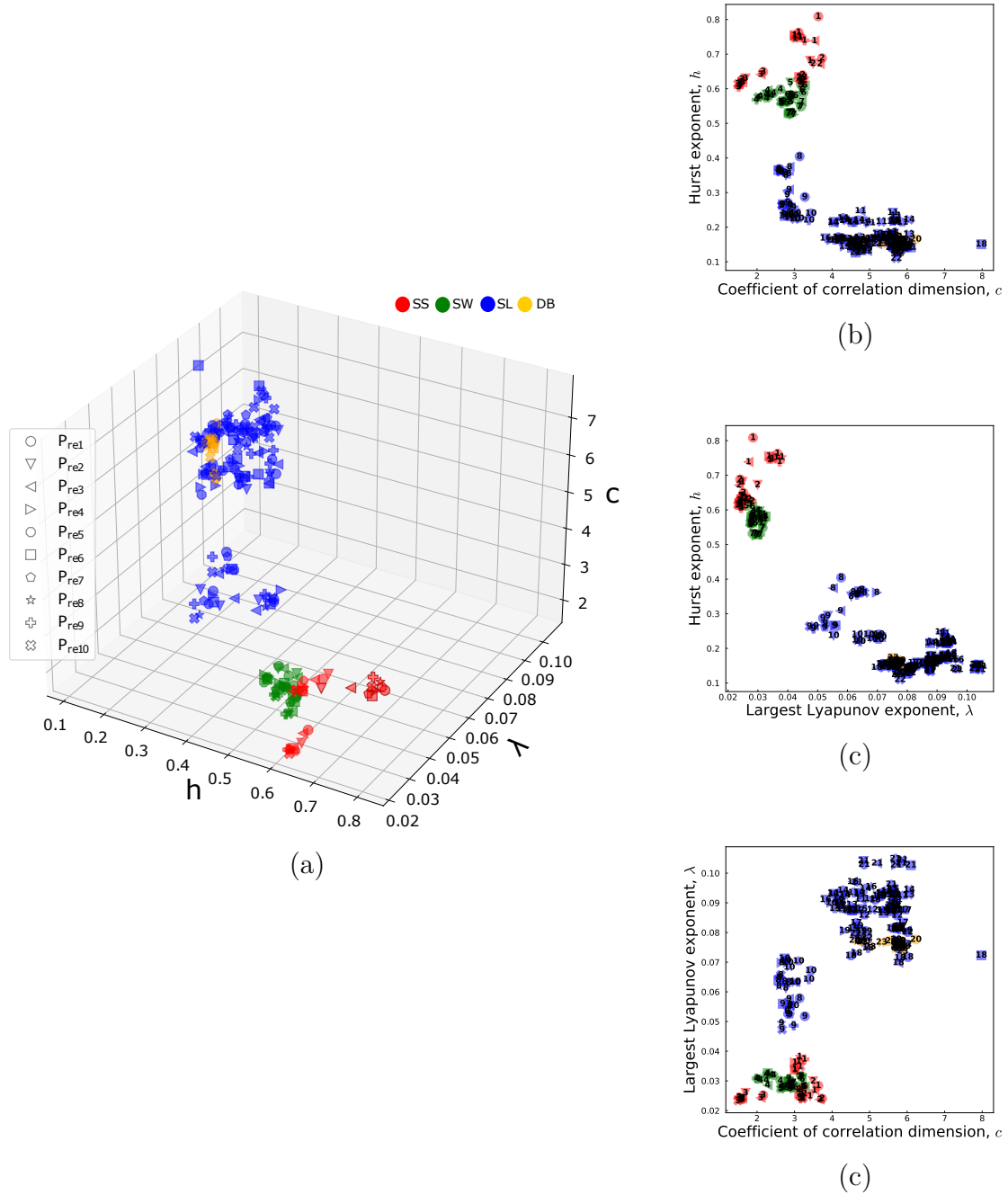


Figure 6.46 – Three-dimensional flow pattern map composed by largest Lyapunov exponent (λ), coefficient of correlation dimension (c) and Hurst exponent (h). The different symbols indicate the sensors from $Pre1$ to $Pre10$. The colours indicate the flow patterns: the red markers are Stratified-Smooth (SS), the green markers are Stratified-Wavy (SW), the blue markers are Slug (SL), and the yellow markers are Dispersed Bubble (DB).

Another important observation is that the data points from the DB pattern, despite being grouped with the SL pattern due to the presence of dispersed bubbles in both, did not exhibit differentiation within the SL pattern. This implies that the technique in question was not effective in distinguishing the DB pattern. However, it is important to emphasise that the technique proved to be effective in distinguishing between intermittent

and stratified patterns.

Tabela 11 – Dimensionless parameters of the largest Lyapunov exponent (λ), coefficient of correlation dimension (c) and Hurst exponent (h), for each experimental point, using P_{re10} pressure sensor.

Point	J_{sl} [m/s]	J_{sg} [m/s]	λ [-]	c [-]	h [-]	Flow pattern [-]
1	0.258	0.106	0.035	3.157	0.749	SS
2	0.259	0.202	0.024	3.221	0.626	SS
3	0.258	0.480	0.023	1.492	0.614	SS
4	0.258	0.899	0.031	2.031	0.576	SW
5	0.258	1.144	0.027	2.633	0.562	SW
6	0.258	1.821	0.029	2.814	0.583	SW
7	0.259	2.036	0.030	2.842	0.530	SW
8	0.580	0.503	0.066	2.635	0.361	SL
9	0.579	0.977	0.047	2.669	0.264	SL
10	0.577	1.431	0.063	3.009	0.224	SL
11	1.191	0.381	0.093	5.316	0.216	SL
12	1.315	0.844	0.088	5.078	0.164	SL
13	1.354	1.402	0.093	6.051	0.179	SL
14	1.301	0.492	0.094	6.056	0.222	SL
15	1.242	1.001	0.094	5.455	0.175	SL
16	1.210	1.219	0.090	5.608	0.169	SL
17	2.150	0.531	0.088	5.700	0.133	SL
18	2.044	1.016	0.075	5.776	0.145	SL
19	2.015	1.184	0.081	4.568	0.143	SL
20	3.154	0.095	0.077	5.819	0.156	DB
21	3.043	0.420	0.104	5.873	0.139	SL
22	2.926	0.855	0.078	5.713	0.110	SL
23	3.328	0.078	0.077	5.322	0.152	DB

6.7 Concluding remarks

This study conducted an analysis of two-phase systems, focusing on the characterisation and identification of flow patterns. Acceleration, pressure, and void fraction data was collected from an experimental test on a 3-inch test pipe, encompassing 23 experimental points across stratified, dispersed bubble, and slug patterns. Initially, the fluid-structure coupling phenomenon was investigated, examining spectrograms and time series of acceleration and pressure signals for various flow patterns. A non-stationary nature is evident in the acceleration time series, indicating the presence of intermittent flow patterns, characterised by the alternation between two distinct structures: the liquid piston and the Taylor bubble, as shown in Figure 6.1. Low-frequency analysis highlights the prevalence of flexural modes in the structure, identified at specific frequencies. Conversely, at higher frequencies, the dominance of cut-on modes in the structure is observed. Additionally, it has been demonstrated that the variation in pressure across the cross-sectional

area, known as pressure gradient, modulates the vibration signal as the unit cell passes through, serving as one of the mechanisms for exciting duct vibrations. Furthermore, through the application of advanced image processing techniques at points associated with the intermittent flow pattern, it was possible to conduct an analysis of the behaviour of small bubbles throughout the entire flow. An expressive increase in the number of bubbles was observed during the liquid piston phase, followed by a sharp decline in the Taylor bubble region. Furthermore, the estimation of the BSD provided a detailed understanding of the system's dynamics. By employing the Rayleigh-Plesset relationship, the vibration resonance in gas bubbles was explored, revealing a broad frequency range that encompasses even the cut-on frequency of circumferential wave modes, as shown in Figure 6.12.

The resonance of the pipe's bending modes and the significant influence of circumferential modes $n = 2$, $n = 3$, and $n = 4$ on the acceleration signal PSDs were observed. Upon analysing the PSD of the pressure signals, a notable increase in amplitude, particularly in the first cut-on mode, is evident. The frequency response function analysis clearly revealed the fluid-structure coupling at the cut-on frequencies, Figure 6.15. The coherence exhibited a significant increase at these cut-on frequencies, highlighting the localised nature of this fluid-structure coupling. Given this, a demodulation method was developed to explore fluid-structure coupling at cut-on frequencies, involving the filtering of the acceleration signal at the cut-on frequency and subsequent envelope extraction through the Hilbert transform. Applying this method revealed that the demodulated acceleration signal accentuates the low-frequency phenomenon associated with the passage of the unit cell. This conclusion was substantiated through a comparative analysis of the PSD of demodulated acceleration signals and void fraction. Thus, by investigating the fluid-structure coupling phenomenon at cut-on frequencies, it was possible to track the Taylor bubble, estimate slug frequency and translational velocity with an error below 15% in most cases, and identify the flow pattern.

A flow pattern classification approach is proposed based on time domain signals obtained from a single accelerometer, exploiting the phenomenon of fluid-structure coupling and using dimensionless thresholds for the Hurst exponent, Pearson and Spearman coefficients. To obtain the results, the acceleration signals were demodulated around the cut-on frequencies. Additionally, the signals were sampled at a frequency of up to 50 Hz, maintaining the duration of the time series at 600 s. The values of the Hurst coefficient indicate persistent behaviour in stratified conditions, characterised by minimal oscillations. On the other hand, intermittent and dispersed bubble patterns exhibit values close to 0.5, suggesting anti-persistent behaviour. Pearson and Spearman correlation coefficients reveal positive associations between demodulated acceleration signals at the two lowest structural cut-on frequencies, showing the ability to distinguish flow patterns, with stratification patterns exhibiting higher correlations compared to dispersed bubble patterns. The three-dimensional representation in Figure 6.39 illustrates distinct clusters corresponding to different flow patterns with groupings of stratified, dispersed bubbles, and slug patterns,

as well as identifying transition points. The presence of transition points between stratified and slug patterns highlights the sensitivity of the proposed method to detect nuances in operational conditions.

Furthermore, this approach was employed in the analysis of field data at a Petrobras refinery. During pipeline vibration measurements using a laser vibrometer, various diameters were considered under diverse operational conditions, including both normal and anomalous scenarios. Despite the challenges faced, the analysis successfully identified flow patterns at each measurement point, as shown in Figure 6.40. However, it is important to note that direct confirmation of the flow patterns was not feasible due to the lack of suitable equipment or specific measurement methods, relying solely on suggestions provided by the plant engineers.

In this study, a system identification analysis was conducted for points in the intermittent pattern. Pressure, pressure gradient, and acceleration envelope signals were used, sampled at a frequency of 50 Hz, with the acceleration signal capturing only the first structural flexural mode. Reconstructed state spaces were visualised using the methods of derivatives, SVD, and time delay embedding. The derivative and SVD methods revealed attractors varying between the Taylor bubble and liquid piston, while the time delay embedding method presented a more irregular structure. The determination of the Lyapunov exponent highlighted the presence of chaotic behaviour, with trajectories exponentially diverging over time. Remarkably, the consistency of Lyapunov exponent and coefficient of correlation dimension values across different measurements suggests that chaotic behaviour is an intrinsic feature of the system, independent of the specific sensor or measurement variable. The comparative analysis between state space reconstruction methods emphasised the importance of approaches such as SVD to mitigate instabilities in estimates. Collectively, these findings contribute to a deeper understanding of the complex dynamic behaviour in intermittent bubble flow systems.

Finally, a methodology was proposed for identifying flow patterns based on pressure measurements, using nondimensional thresholds such as the Hurst exponent, correlation dimension coefficient, and Lyapunov exponent. All experimental data points from sensors and pressure measurements were employed, totalling 230 data series. The analysis of the Hurst exponent on pressure sensors revealed persistent series for stratified patterns and anti-persistent series for intermittent patterns. The Lyapunov exponent analysis confirmed chaotic behaviour in all patterns, with turbulence and complexity present. Despite variations in values, both stratified and intermittent patterns exhibited chaotic behaviour. The correlation dimension coefficient highlighted distinctions in complexity among flow patterns, contributing to effective differentiation between them. The three nondimensional parameters - Hurst exponent (h), Lyapunov exponent (λ), and correlation dimension coefficient (c) - were calculated for all sensors and test points, resulting in distinct groupings based on flow patterns, as illustrated in Figure 6.46. Notably, the smooth and wavy stratified patterns grouped together, distant from the slug pattern. The

technique demonstrated effectiveness in differentiating between intermittent and stratified patterns, although it faced challenges in distinguishing dispersed bubbles within the slug pattern. Overall, the methodology presented promising results in the classification and understanding of complex flow patterns.

7 Conclusions

7.1 General conclusions

This thesis investigated the application of indirect approaches to identify and characterize flow patterns in two-phase liquid-gas flow, using non-invasive sensors. The advantages of these approaches include eliminating the subjectivity inherent in traditional methodologies, cost-effectiveness, and easy integration with supervisory systems and the Internet of Things. The proposed methodologies used signal processing techniques and physical models, employing non-intrusive pressure and acceleration sensors in horizontal pipe flow.

The Segment Anything Model (SAM) was employed for image processing to analyse the bubble diameter spectrum in two-phase flow images. SAM, a deep learning tool, excelled in the precise segmentation of bubbles, enabling the extraction of detailed information about their sizes. The corresponding Rayleigh-Plesset frequencies distribution were also calculated and they present a spectral distribution over all the frequency band of interest for the structural excitation. In addition, the timed-domain modulation of the number and distribution of bubbles plays a role on the pipe vibration response.

It is experimentally shown the pressure and structural coupling due to wave mode veering and coupling, similar to what is found for single-phase flow. Although the flow alters the cut-on frequencies, the frequency band of the coupling can be estimated from and an analytical expression derived for empty pipes due to the typical low flow velocities of the intermittent pattern. Moreover, the coupling was investigated as a local phenomena, as indicated by values of the coherence function. It is also shown that the PSD peak of the demodulated acceleration response from the cut-on frequency matches that of the void fraction measurements for both cases. This indicates that this measurement can be used to probe on the void fraction of an intermittent flow in horizontal pipes. This results paves the way for new solutions for non-intrusive measurement of features of two-phase flow.

Frequency analysis of the demodulated vibration signal showed that the vibration is modulated at low frequency by the void fraction oscillations in an intermittent pattern, caused by the transition between liquid piston and Taylor bubble. Consequently, one can estimate the frequency of slugs with errors of less than 15% for most points. Furthermore,

using two demodulated stations, it was possible to successfully estimate the Taylor bubble velocity using cross-correlation and windowing functions. Finally, some features were extracted from the demodulated measurements, such as Hurst exponent, Spearman and Pearson coefficient, in order to generate a three-dimensional flow map. These parameters were able to separate each flow pattern into different regions, that can be easily used as an indirect flow pattern identification tool in real industrial systems.

In the present investigation, a non-linear system identification analysis was carried out for experimental points in the intermittent pattern. Pressure, pressure gradient and acceleration signals were used, sampled at a frequency of 50 Hz. It is essential to note that the acceleration signal specifically captured the first mode of structural bending. Through the analysis of reconstructed state spaces and chaos metrics, it was possible to characterise the intrinsic dynamics of the system. The embedding dimension analysis revealed that, for most experimental points, the system has an intrinsic dimension of approximately $d_a = 3$. This suggests that the complexity of the system is related to three main directions in phase space, regardless of the type of signal considered (pressure, pressure gradient, or acceleration).

The observed agreement between the results of Lyapunov exponents and the coefficient of correlation dimensions in slug flow patterns, derived from different signals such as acceleration, pressure, and pressure gradient, constitutes a significant discovery in the context of this study. Lyapunov exponents and correlation dimensions serve as robust metrics aimed at evaluating the chaotic behaviour and complexity of the system. The observed agreement between these metrics across different types of signals within the same system underscores the stability and universality inherent in the system's dynamics, thus reinforcing the consistency of these quantities throughout the measurements. This emphasises the relevance of chaos metrics as fundamental tools in characterising and studying highly complex dynamic systems.

Through the analysis of pressure signals subsampled at 50 Hz, we identified flow patterns using metrics such as the Hurst exponent, correlation dimension coefficient, and Lyapunov exponent. The results of the Hurst exponent revealed that stratified patterns had values between 1 and 0.5, indicating persistence, while intermittent patterns had Hurst exponent values between 0 and 0.5, indicating anti-persistence. This difference is related to the oscillatory nature of intermittent patterns, alternating between the Taylor bubble and the liquid piston.

The results of the Lyapunov exponent indicated chaotic behaviour in all flow patterns, with positive values denoting turbulence and complexity. Stratified patterns had lower Lyapunov exponent values compared to intermittent patterns, reflecting differences in trajectory divergence rates due to increased turbulence in the latter. The correlation dimension coefficient revealed the complexity of reconstructed attractors, with stratified patterns exhibiting low complexity reflected in low correlation dimension coefficient values compared to other patterns.

Furthermore, the analysis of the Hurst, Lyapunov, and correlation dimension coefficients allowed for the formation of clusters, distinctly identifying stratified and slug patterns. Furthermore, low-frequency slug patterns, which indicate a higher incidence of the stratified region due to the longer length of the Taylor bubble, were positioned between these clusters. The dispersed bubble patterns were closer to the slug patterns due to the presence of dispersed bubbles. This phenomenon highlights that the methodology is capable of capturing the physical parameters of the system that are inherent to each pattern, proving to be a powerful and useful thresholding technique. While the technique is efficient in distinguishing between stratified and intermittent patterns, no clustering was observed for dispersed bubble patterns. Further studies should be conducted, such as calculating the Kolmogorov entropy, since it quantifies the complexity and unpredictability of a chaotic dynamical system, indicating the rate of information loss as the system evolves over time.

In this study, the application of non-invasive approaches for the identification and characterisation of patterns in two-phase flow proved highly effective. The investigation of fluid-structure coupling highlighted coherence in the cut-on frequency regions, providing crucial insights for understanding intermittent patterns. Additionally, chaos metrics applied to time series enabled the identification of chaotic behaviours in intermittent flow patterns. The proposed methodology, extending to pressure analysis, demonstrated to be a robust tool for evaluating complex systems. In summary, this study not only advances theoretical understanding but also offers practical and innovative solutions for non-intrusive measurements in two-phase flows.

7.2 Summary of the main findings

Based on the proposed analysis, the main finding so far can be given as,

- Fluid-Structure coupling: the study reveals pressure and structural coupling due to wave mode veering and gyroscopic coupling, offering insights into the local phenomena and providing a foundation for non-intrusive measurement solutions.
- Characterisation from acceleration signals: from the demodulated vibration signals, it is possible to track the Taylor bubble and estimate the slug frequency and Taylor bubble velocity.
- Pattern identification from acceleration signals: from the demodulation vibration signal it is possible to identify the flow pattern using dimensionless thresholds of the Hurst exponent and the Pearson and Spearman coefficients, using only an acceleration sensor.
- Pattern identification from pressure signals: from the pressure signals, it is possible to differentiate between intermittent and stratified flow patterns using dimension-

less thresholds of Hurst exponent, Lyapunov exponent, and correlation dimension coefficient, using only a pressure sensor.

7.3 Future work recommendations

In this section suggestions for further research are proposed.

- To investigate other fluid-structure coupling regions, such as the range between 200 Hz and 1000 Hz, has proven to be a region with high coherence, indicating the importance of a more in-depth analysis in this range to understand the coupling patterns and associated dynamics.
- To relate the bubble size distribution in two-phase flow to the turbulence dissipation rate. The bubble shape and bubble size spectrum are inherently connected to how turbulence manifests in the system.
- To apply the developed methodology to estimate flow patterns from pressure and acceleration signals to include additional patterns, such as elongated bubble and annular, which were not achieved with the current experimental setup. Furthermore, it is crucial to conduct analyses considering different pipe diameters and liquid viscosity.
- To develop a classifier using the dimensionless thresholds through the analysis of acceleration and pressure signals, based on machine learning algorithms such as K-Nearest Neighbours (KNN) or Support Vector Machines (SVM). The thresholds explored in this study, including Hurst exponent, Lyapunov, Spearman, Pearson, and correlation dimension coefficient, have the ability to capture the inherent complexity and dynamics associated with each flow pattern, thereby making the classifier more accurate and robust.
- To analyse the pressure time series in terms of Kolmogorov entropy, which can provide important information about the system's dynamics. By employing concepts from turbulence theory, Kolmogorov entropy offers a robust measure of the complexity and irregularity of the flow, allowing for a detailed assessment of the dynamics of two-phase liquid-gas flow.
- To evaluate the identification of flow patterns using vibration sensors in field data and compare the results with commercially established meters, especially in Exploration & Production contexts.

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Appendix

A Void fraction meter

This appendix is based on the paper "Development and dynamic characterisation of a conductance-based void meter array" by **Das Neves, D.A.**; Vieira, S.C.; Cenzi, J.R.; Barbosa, B.R.; Dos Santo, J.N.; Castro, M.S.; Fabro, A.T. submitted for publication and currently under review at Measurements.

A.1 Development and dynamic characterisation of a conductance-based void meter array

Conductance-based void meters are widely used for monitoring two-phase flow in pipes. In this appendix, a new measurement array is developed in which the cross-talking issue is addressed. For this new configuration, a demodulation algorithm is proposed and a robust static calibration scheme is developed, ensuring the same statistical properties in all sensors of the array. An experimental setup for dynamic characterisation is proposed, with controlled inlet and high-speed camera measurements. It is shown that the sensors present linear phase and constant gain in the frequency bandwidth of the input step from the experimental bench. The sensor array is validated in a second experimental bench, capable of producing two-phase for several flow condition and for stratified, intermittent, and dispersed bubble flow patterns, and it has shown excellent agreement with the standard techniques. The proposed sensor array is relevant for application in which the void fraction dynamics must be well characterised.

A.2 Introduction

Two-phase gas-liquid flows in pipelines are encountered in various industrial applications, notably in the oil exploration and production industry. Monitoring two-phase flow rates in pilot plants under controlled conditions holds significant importance for a better understanding of the underlying physical phenomena. This understanding, in turn, is employed for the validation and enhancement of existing flow models and the development of new measuring instruments. In this context, it is essential to emphasise that intrusive sensors, such as resistive and Wire Mesh Sensors (WMS), play a pivotal role

in laboratory-controlled systems (Velasco Peña; RODRIGUEZ, 2015). They are employed to investigate critical variables related to two-phase flow, including, but not limited to void fraction, bubble velocity, and bubble size (SANTOS et al., 2019; RODRIGUES et al., 2020; AL-YAHIA et al., 2023; VIEIRA et al., 2023). The use of these intrusive sensors plays a crucial role in validating proposed theoretical models, thereby enabling continuous improvement in the understanding and effectiveness of measurement techniques used in the industry.

These void fraction sensors consist of a transmitter wire and a receiver wire, which are perpendicular to each other. An oscillating voltage or current is applied to the transmitter wire, so current flows from the transmitter wire to the receiver wire through a sensitive volume around the crossing point of the two wires. According to the current received by the wires of the receiver, the conductivity or permittivity of the sensitive volume can be measured (PRASSER; BÖTTGER; ZSCHAU, 1998). However, a residual electrode polarisation effect may occur, especially if the electrode impedances are not completely balanced. Thus, the parasitic impedance effect in the system generates the cross-talking effect between different sensor pairs and between the electrodes and each sensor, consequently, the local void fraction measurements would contain a systematic error (DIAS et al., ; ZHANG; XIAO; GU, 2019). Few authors investigate the relationship between cross-talking and the void fraction measurement, only stating the switching between pairs of sensors as an alternative to eliminate the effect (Velasco Peña; RODRIGUEZ, 2015; LIU; TAN; DONG, 2015; RODRIGUES et al., 2020).

Prasser et al. (PRASSER; BÖTTGER; ZSCHAU, 1998) propose a methodology to reduce cross-talk by lowering the impedance of transmitter outputs and receiver inputs. This prevents the potential difference between inactive electrodes. However, the author did not effectively assess the presence of cross-talk in the results. Additionally, there was no evaluation of the sensor's dynamic response or a detailed explanation of the signal acquisition procedure. Dos Santos et al. (SANTOS et al., 2019) investigates the measurement principle of dual-wire conductance sensors, where the electronics employ analog switches to generate a 10 kHz square wave excitation with a bipolar voltage. The author states that this is done to mitigate electrolysis effects and interference, but does not assess the method's efficiency or provide details on the demodulation process. Furthermore, the authors describe the estimation of the rise and fall time of the sensor circuit, as well as its operating range and frequency response to a step input. However, it is important to note that the time delay measurement methodology considers only the response of the electronic circuit, not involving the sensing element. Zhai and Meng (ZHAI; YANG; MENG, 2019) proposed a measurement system employing a 8×8 conductance Wire Mesh Sensor (WMS) with sinusoidal voltage excitation at 20 kHz. It utilises an analog switch array controlled by a microcontroller to connect transmitter wires sequentially. Phase distribution information is collected and processed using precision amplifiers (AD620) and a wide-bandwidth rms-to-dc converter chip (AD637). The data acquisition system

records phase distribution across the pipe cross-section, allowing dynamic monitoring. The study does not evaluate the technique’s efficacy. This study, as per the provided description, does not assess the effectiveness of the technique and offers limited insight into the demodulation process. This deficiency becomes apparent when examining the generated temporal signal, which exhibits significant discretization and modulation at a much lower frequency, raising concerns about the quality of the process.

In summary, several authors employ sensor conductance-based measurement (SILVA; SCHLEICHER; HAMPEL, 2007; RODRIGUEZ et al., 2015; ZHANG; XIAO; GU, 2019; MERIBOUT et al., 2020; AL-YAHIA et al., 2023), however, there is a gap in the literature regarding the evaluation of cross-talk in these systems, when there is more than one sensor used simultaneously, and the analysis of the sensor’s dynamic response. In conclusion, while these studies provide valuable information, they do not fully validate and detail the performance of these sensors. Therefore, further research and assessments are required to enhance their reliability and practicality. With the aim of addressing this gap, this paper investigates the dynamic response of the resistive sensor in terms of time and frequency, assesses the effect of cross-talk on the measurements, and proposes a new signal demodulation methodology. So, this paper aims to investigate the dynamic response of a conductance-based void meter array with four sensors. The basic sensing elements are presented in Section A.3. Two different circuits are used for measuring the sensing element conductance, presented in Section A.4. In the first, that is typically used, the sensors in the array share the same sinusoidal source and the outputs are measured simultaneously by a voltage divider. In the second, two multiplexer switches are used to switch both the receiver and the transmitter in the array, such that only one pair is active at the same time. The calibration method for the array is presented in Section ??, based on Lagrange multipliers, such that the measurements at each sensor in the array have the same statistical properties. Section A.6 briefly reviews two approaches for the dynamic characterisation of the sensor array. Section A.7 presents two experimental setups. The first is to generate a step input response of the sensor and the second is a typical two-phase flow bench, generating different flow patterns. Section ?? presents the experimental sensor response in the non-switching and switching configurations. It is shown that the non-switching configuration, typically used, presents cross-talking, which is avoided by multiplexing the sensors in the array. Moreover, Section A.9 presents the experimental results of the sensor dynamics in terms of time response, frequency range and linear phase. Finally, Section A.11 presents the concluding remarks.

A.3 Basic sensing elements

In this section, a brief overview of the basic sensing element for conductance-based measurement of void fraction is presented. An indirect approach to measuring the void fraction is by measuring the height of the liquid with a subsequent non-linear relation to

the void fraction of the cross-sectional area. The basic principle of a conductance void meter consists of implementing a voltage divider, in which one of the impedances is given by the sensing element in the two-phase flow and the other is a reference resistor for output measurement (SONG; CHUNG; NO, 1998). The sensing element can be designed as a two-wire sensor, which consists of two stainless steel parallel wires distanced by dw , one functioning as a transmitter and the other as a receiver, as shown in the Figure A.1 (a). The electronics used for effectively extracting this information is presented in the next section. This device measures the height of the liquid through electrical conductance measurements between the two wires. Since gas and liquid have different electrical conductivity and/or permittiveness, the conductance obtained is a measure of the liquid content between the wires. A sensor array can be assembled with several of these boards evenly spaced at a distance dy apart, as shown in Figure A.1(b) with four sensor boards.

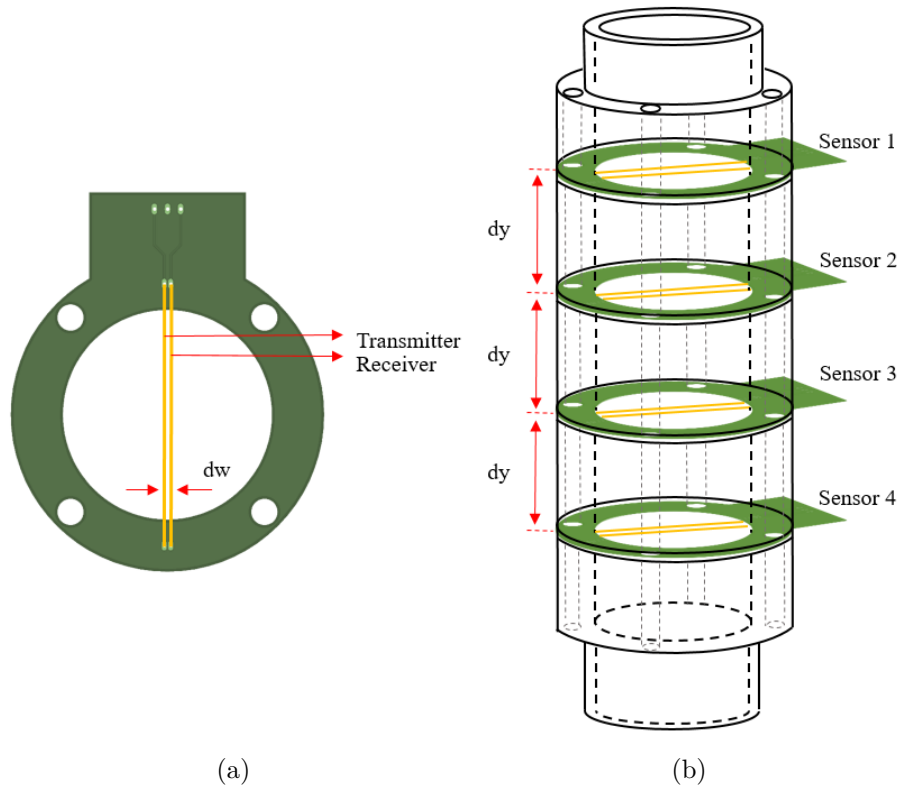


Figura A.1 – Illustration of resistive sensor board with transmitter and receiver wires distanced by d_w (a). Sensor array with 4 boards distanced by dy from each other (b).

In Figure A.2 (b), two wires are immersed in water at which the potential difference produces an impedance Z_1 for the sensor 1 and Z_2 , Z_3 and Z_4 for the sine 1, 2, 3 and 4 respectively. Typically, circuits for conductance measurement of fluids are excited by dc-free sinusoidal source where an operational amplifier at the output acts as a current-to-voltage converter, i.e., the output voltage is proportional to the current through the two-phase

liquid flow, and the circuit gain is given in terms of (SHI et al., 2021),

$$|V_o| = \left(\frac{1}{G_f} \right) G_m |V_i|, \quad (\text{A.1})$$

where V_o and V_i are the voltage output and input, G_f is the reference conductance and G_m is the two-phase liquid flow conductance. An alternating signal excitation should be typically chosen at the kHz range to avoid polarisation and enhanced ageing effects at electrodes or a capacitive dominant equivalent circuit (SHI et al., 2021). A demodulation of the output signal is required to measure the time change in the conductance G_m .

Furthermore, the interaction between subsequent sensors in the array is important and can produce undesired or parasitic impedances Z_{12} , Z_{23} , and Z_{34} , which are directly dependent on the distance between the sensors, as schematically represented in Figure A.2 (b). This effect can cause interference in the sensing element and consequent spurious measurements, as known as cross-talking.

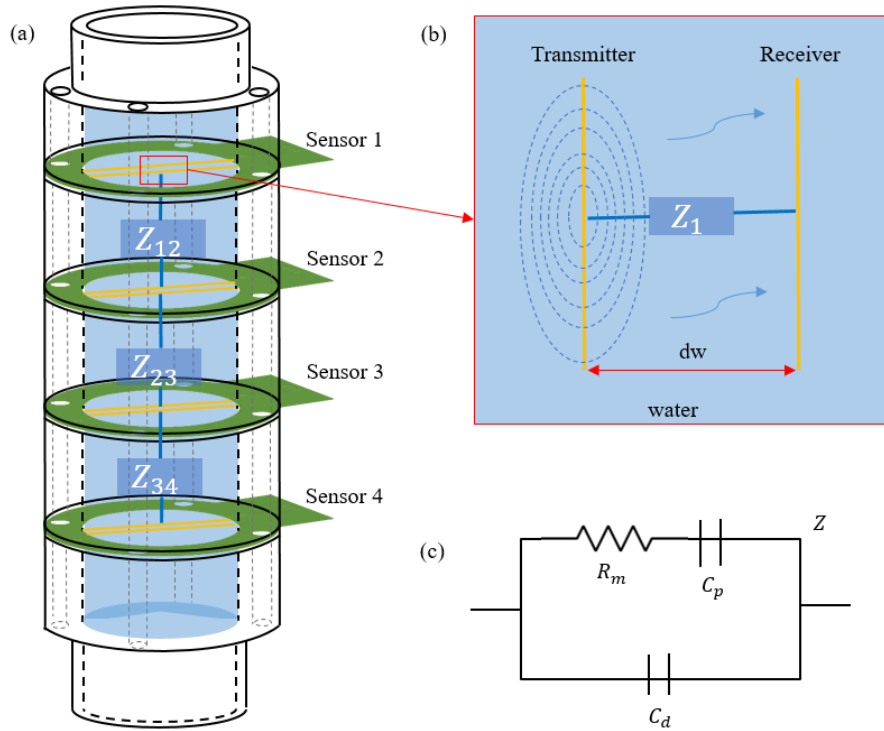


Figure A.2 – Illustration of the resistive sensor with 4 boards and immersed in water (a), the parasitic impedances between the boards are represented by Z_{12} , Z_{23} , and Z_{34} . The parasitic impedance between the receiver and the emitter wires (b) is represented by Z_1 . Electronic circuit with the representation of parasitic impedance (c) (SHI et al., 2021).

Equivalent impedance of a typical measurement setup for fluid monitoring, consisting of two electrodes, where R_m is the resistance that reflects the measured fluid's conductivity, C_d is capacitance that reflects the fluid's dielectric constant, and C_p is the parasitic capacitance caused by the polarization near electrodes, as shown in the

Figure A.2 (c). Therefore, the impedance of the measured fluid between two electrodes can be expressed as (SONG; CHUNG; NO, 1998),

$$Z(\omega) = \left(\frac{1}{R_m + \frac{1}{i\omega C_p}} + i\omega C_d \right)^{-1}. \quad (\text{A.2})$$

In most applications, the reactance is neglected and conductance or resistance is obtained based on ohm's law. Thus considering only the resistance measurements to assess the phase fractions. But, the existence of the conductive medium will make the mutual inductance to become complex valued. To solve the circuit and obtain the void fraction value comprised, a new approach is proposed based on switching the sensors.

A.4 Modulation and demodulation methods

The source of information or electrical impedance produced by the sensor wires is amplified by an amplifier circuit and modulated by a carrier, alternating waveforms. The interface to the communication channel is the two-phase medium within the pipeline. At the other end, there is a demodulator where the main objective is to recover the original information modulated in the receiver. In this section, the method of modulation and demodulation with switching and non-switching approaches will be presented along with the electronic circuit used in each system.

A.4.1 No-switching method

Figure A.4 shows a typical, or non-switching circuit, for direct impedance measurement of the 4 sensors, in which an alternating voltage is applied to the excitation electrode and the current due to the (virtually) grounded receiver electrode is measured. The electronics apply an alternating voltage to the excitation electrode using a function generator to create a sine wave of 5 Vpp peaks with a frequency of 3.2 kHz. Prior to the excitation, the signal is buffered through an amp-op (Texas Instruments TL072), effectively providing a low output impedance. In the receiver, an amplifier circuit of the integrator amplifier type with negative feedback through a capacitor in parallel with a resistor. In this application, a potentiometer was used to adjust the amplifier gain for each sensor independently. The receiver signal is applied through an input resistor to the inverting input of this op-amp (Texas Instruments TL072) and the non-inverting input is grounded. Finally, the amplifier output signals (*out1*) are measured in data acquisition systems (NI 9201). This circuit is replicated for all the other sensors in the array, leading to the corresponding output signals (*out2*, *out3* and *out4*)

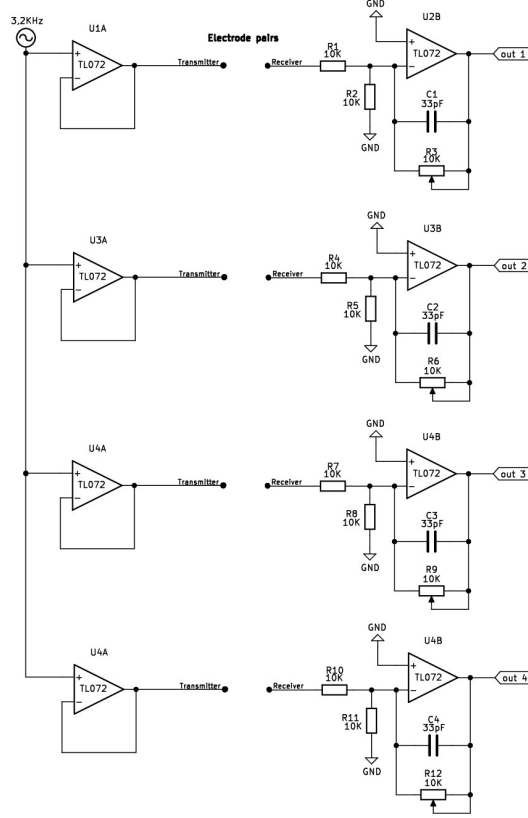


Figura A.3 – Electronic circuit for the resistive sensor for the no-switching approach, modified from Dos Santos et al. (SANTOS et al., 2019). On the left side, there is the input to the system with an oscillating signal and a buffer connected to the transmitting wire of the resistive sensor. On the right side, the amplification circuit is connected to the receiving wire of the resistive sensor and then captured by the acquisition system.

Modulation involves the forward translation of the message spectrum, while demodulation involves the inverse translation, facilitating the retrieval of the original message embedded within the modulated signal (PROAKIS; SALEHI, 2008). In this context, liquid height information is encoded through amplitude modulation (AM), wherein the carrier signal, for instance, can take the form of a sine wave or square wave. In the system under consideration, the information waveform modulates the carrier's amplitude, thereby determining the envelope of the modulated carrier. Subsequently, considering the liquid height information as the baseband signal denoted by $h(t)$, and $q(t) = A_q \cos(2\pi f_q t)$ as a sinusoidal carrier, where f_q and A_q represent the carrier's frequency and amplitude, respectively. The amplitude-modulated output is expressed as

$$s(t) = (1 + h(t))A_q \cos(2\pi f_q t), \quad (\text{A.3})$$

where $h(t) = A_m \cos(2\pi f_m t)$, f_m is an information frequency or modulating frequency, which is much smaller than the carrier frequency ($f_m \ll f_q$). This expression can be

rewritten as

$$s(t) = A_q \cos(2\pi f_q t) + \frac{1}{2} A_q A_m \cos[2\pi(f_q + f_m)t] + \frac{1}{2} A_q A_m \cos[2\pi(f_q - f_m)t]. \quad (\text{A.4})$$

Demodulation is performed in the receiver to downshift the frequency content back to its baseband frequency. There are two types of demodulation of the AM signals: synchronous demodulation and non-synchronous demodulation. In synchronous demodulation, the receiver must generate a carrier signal synchronised in phase and frequency, which is not the case with the system in this work. However, the carrier signal is not needed in asynchronous demodulation, this is accomplished by the use of a band-pass filter that is tuned to the appropriate carrier frequency. Figure A.4 shows the conceptual model of the AM receiver. As the signal enters the receiver, it passes through the band-pass filter, which is tuned to the f_q . Next, the recovered signal is passed through an envelope detector to recover the original signal that was transmitted.

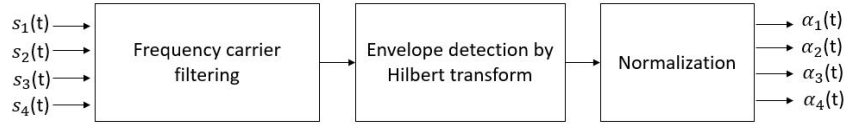


Figura A.4 – Amplitude demodulation process: filtering at carrier frequency and envelope detection. Note that the input signals of the four resistive sensors are independent, leading to four equally independent output signals.

A.4.2 Switching method

Figure A.5 shows switching circuit for direct impedance measurement of the four sensors. An alternating voltage is applied to the excitation electrode and the current is measured at the receiving electrode. Multiplex circuits are used in electrical resistance tomography, for instance, in order to share the current source and voltage measurement stages between any number of electrodes, despite of longer acquisition time and phase delay (DICKIN; WANG, 1996; SHI et al., 2021). Two multiplexer switches (CD 4052) are used to switch both the receiver and the transmitter of the array, so only one pair is activated per cycle. This is used to avoid electrolysis effects and the excitation scheme prevents the sensor from being simultaneously enabled, thus avoiding cross-talk, as discussed in Section A.3. The control of this switching is performed by two digital output signals (D_2 and D_3) from an Arduino Uno (ATmega328 microcontroller). Finally, similar to non-switched configuration, the outputs of the receivers are amplified. Table 12 shows the activation combination of each pair of sensors built from the digital outputs D_2 and D_3 , from which it is possible to control 4 pairs of sensors by synchronising them in a square wave with the corresponding activation sequence. When D_2 and D_3 are set to zero, the transmitter and receiver of the R_1 board are active and the rest of the array is inactive,

and this is repeated for the other combinations. Also, it is possible to observe that, unlike the non-switched circuit, the output of the switched system is unique, i.e., the separation of the signals from the sensors from each sensor and consequent recovery of information is performed using the synchronised square wave, also following Table 12.

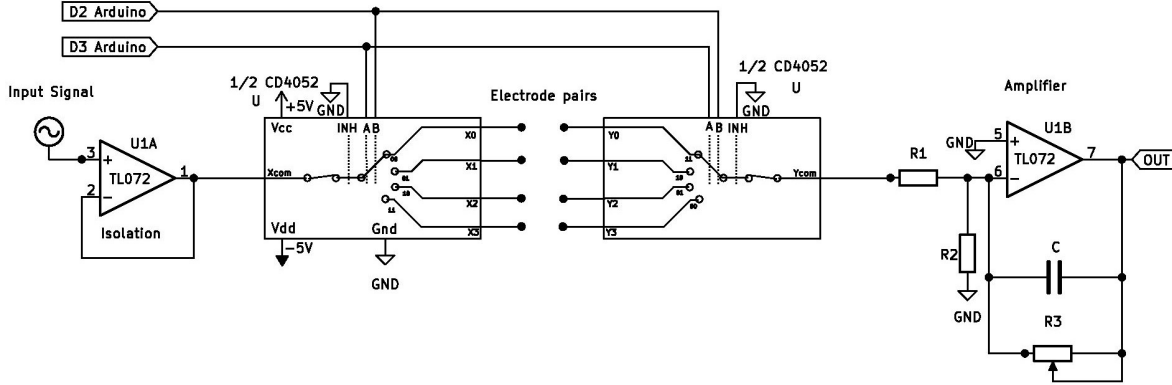


Figura A.5 – Electronic circuit for the switching approach. On the left side, there is the oscillatory input of the system feeding a multiplexer switch, connected to the transmitter wire of the sensor. On the right side, there is another multiplexer switch and the amplification circuit connecting the circuit to the receiver wire. Both multiplexer switches are controlled by digital inputs D_2 and D_3 and Table 12 shows the activation table and each sensor.

Tabela 12 – Digital output for activation each electrode pairs in a resistive sensor array with four sensor boards.

Sensor	D_2	D_3
R_1	0	0
R_2	0	1
R_3	1	0
R_4	1	1

The electronic circuit in Figure A.5 shows the modulation process of the electrical conductivity signal. For each station, the electrical conductivity signal is multiplied by the carrier $q(t)$. On the receiver side, after the amplifier, there is a single output containing the information of the four stations. It is important to highlight that the switching frequency f_{CH} must be lower than the carrier frequency ($f_{CH} \ll f_c$). In addition, imposing f_{CH} and f_c as integers multiples, so each switching cycle allocates an integer number of carrier wave periods, significantly reduces leakage. Furthermore, the acquisition frequency f_s must be at least twice the carrier frequency ($f_s > 2f_c$) according to the Nyquist rate.

The demodulation of the output signal from the switching approach $s(t)$ consists of performing a separation of the signals from each sensor, following the synchronised square wave produced by D_1 and D_2 , as shown in the first part of Figure A.6. Subsequently, for the j^{th} cycle of the switching, a time series with an integer number of periods of the

carrier wave $q(t)$ is available for the i^{th} sensor. From this point, the sensor response can be characterised by the gain and phase difference with respect to the input carrier wave.

In practice, the carrier wave is a noisy component and might present slight amplitude variations during the measurement, thus a linear system identification approach is proposed. Denoting $q_i^j(t)$ and $s_i^j(t)$ as the carrier and modulated signals at the j^{th} cycle of the switching and the i^{th} sensor, respectively, they can be defined as the input/output signals of a linear system $G_i^j(f)$ at the frequency f_c of the carrier signal, such that

$$S_{qqi}^j(f_c) = |G_i^j(f_c)|^2 S_{SSi}^j(f_c), \quad (\text{A.5})$$

where $S_{qqi}^j(f_c)$ and $S_{SSi}^j(f_c)$ are the Power Spectral Density (PSD) of $q_i^j(t)$ and $s_i^j(t)$, respectively. Notice that this identification problem is defined only for a single frequency component f_c . Consequently, by integrating both sides of Equation A.5 and using a fundamental propriety of the PSD yields $(\sigma_{qqi}^j)^2 = |G_i^j(f_c)|^2 (\sigma_{SSi}^j)^2$. Notice that this relation is valid when $q_i^j(t)$ and $s_i^j(t)$ are zero-mean signals, which can be enforced with no loss of generality. Moreover, a Hamming window is applied at both $q_i^j(t)$ and $s_i^j(t)$ to further reduce leakage. Consequently, an expression for efficient calculation of the gain $|G_i^j(f_c)| = G_i^j$ of the i^{th} sensor at the time sample j^{th} is given as,

$$G_i^j = \frac{\sigma_{SSi}^j}{\sigma_{qqi}^j}. \quad (\text{A.6})$$

The phase difference can also be calculated by this approach but using the cross-correlation time delay between $q_i^j(t)$ and $s_i^j(t)$. Finally, it is important to highlight that the resulting time series has sampling frequency f_{CH} .

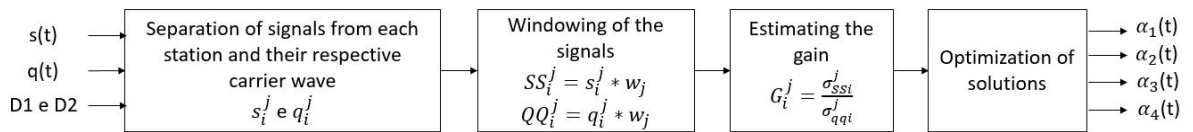


Figura A.6 – Switching demodulation process: separation of the input signal using binary words, followed by signal windowing, then gain estimation, and finally signal optimization.

A.5 Calibration of the sensor array

In this section, a calibration procedure for the sensor array is proposed. The approach is robust in the sense that the sensor array is calibrated such that the measurements at each sensor have the same statistical properties. For each sensor in the array, a linear model is assumed relating the impedance G_i^j to the liquid height h_i^j , such that $h_i^j = a_i G_i^j + b_i$ (SANTOS et al., 2019), which $(\cdot)_i^j$ stands for the quantity h or G at the j^{th} time sample

with up to N points from the i^{th} sensor. Thus, the static calibration problem can be cast as finding a_i and b_i for the i^{th} sensor that minimises the square error defined as

$$\varepsilon^2 = (\mathbf{Y} - \mathbf{X}\beta)^T (\mathbf{Y} - \mathbf{X}\beta), \quad (\text{A.7})$$

where $\mathbf{Y} = [\mathbf{h}_1^T \cdots \mathbf{h}_2^T \cdots \mathbf{h}_3^T \cdots \mathbf{h}_4^T]^T$, in which $\mathbf{h}_i$ stands for the liquid height measurement at the i^{th} sensor, $\beta = [a_1 \ b_1 \ a_2 \ b_2 \ a_3 \ b_3 \ a_4 \ b_4]^T$ is the vector containing the coefficients a_i , b_i for each sensor in the array, and $\mathbf{X}$ is the matrix with the corresponding the linear model of the sensor.

Given the stochastic nature of the two-phase flow, it is also reasonable to constrain the problem such that the liquid height from all sensors in the array have the same mean value $\mu(\mathbf{h}_i)$ and variance $\sigma^2(\mathbf{h}_i)$, following that all signals are delayed measurements of the same propagating wave (VIEIRA et al., 2023). From the sensor linear model, it follows that $\mu(\mathbf{h}_i) = a_i\mu(\mathbf{G}_i) + b_i$ and $\sigma^2(\mathbf{h}_i) = a_i^2\sigma^2(\mathbf{G}_i)$. Subsequently, this restriction can be rewritten for the sensor array as,

$$\mathbf{L}\beta = \mathbf{r}, \quad (\text{A.8})$$

where $\mathbf{r} = [0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$ and,

$$\mathbf{L} = \begin{bmatrix} \sigma(\mathbf{h}_1) & 0 & -\sigma(\mathbf{h}_2) & 0 & 0 & 0 & 0 & 0 \\ \mu(\mathbf{h}_1) & 1 & -\mu(\mathbf{h}_2) & -1 & 0 & 0 & 0 & 0 \\ \sigma(\mathbf{h}_1) & 0 & 0 & 0 & -\sigma(\mathbf{h}_3) & 0 & 0 & 0 \\ \mu(\mathbf{h}_1) & 1 & 0 & 0 & -\mu(\mathbf{h}_3) & -1 & 0 & 0 \\ \sigma(\mathbf{h}_1) & 0 & 0 & 0 & 0 & 0 & -\sigma(\mathbf{h}_4) & 0 \\ \mu(\mathbf{h}_1) & 1 & 0 & 0 & 0 & 0 & -\mu(\mathbf{h}_4) & -1 \end{bmatrix}. \quad (\text{A.9})$$

This constrained minimisation problem can be approached by Lagrange multipliers generalised under the Karush-Kuhn-Tucker (KKT) conditions (NOCEDAL; WRIGHT, 2006) as,

$$\mathcal{L}(\beta, \lambda) = (\mathbf{Y} - \mathbf{X}\beta)^T (\mathbf{Y} - \mathbf{X}\beta) + 2\lambda^T (\mathbf{L}\beta - \mathbf{r}), \quad (\text{A.10})$$

where λ is the vector of Lagrange multipliers. Subsequently, imposing that $\nabla_{\beta}\mathcal{L} = 0$ and $\nabla_{\lambda}\mathcal{L} = 0$, where $\nabla_{\beta, \lambda}$ stands for the gradient with respect to the vector β and λ , respectively, yields the linear system

$$\begin{bmatrix} \mathbf{X}^T\mathbf{X} & \mathbf{L}^T \\ \mathbf{L} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \beta \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{X}^T\mathbf{Y} \\ \mathbf{r} \end{bmatrix}, \quad (\text{A.11})$$

whose solution gives the coefficients a_i and b_i of each sensor at β . Notice that the ordinary least-square problem is a particular case of when λ is a null vector.

Finally, the void fraction is obtained by the geometric relation for a pipe with circular cross-section as

$$\alpha_i^j = 1 - \left[r^2 \cos^{-1} \left(\frac{r - h_i^j}{r} \right) - (r - h_i^j) \sqrt{2rh_i^j - (h_i^j)^2} \right] \frac{1}{A_T}, \quad (\text{A.12})$$

where r is the radius of pipe and A_T is the pipe cross-sectional area.

A.6 Dynamic characterisation

In this section, the dynamic response of the sensor is characterised through an experiment in which a unit step input is employed, as will be detailed in the following section. The sensing element can be modelled as a first-order system and its time response to the step input can be utilised as a measure of its frequency bandwidth. Under this assumption, an input/output linear time-invariant system identification approach can be used to estimate the sensor gain and phase. Given that a first-order system exhibits a delayed response to an abrupt change in the input, manifesting itself only after a characteristic time constant denoted as τ , the rise time t_R is defined as the time interval that separates two specific points along the ascending portion of the signal output when subjected to an input step function. Typically, this measurement is taken between 10% and 90% of the signal's total value. Consequently, it is feasible to compute the system's response time within the range of 10% to 90% of its value as $t_{0.1} = -\tau \ln(1 - 10)$ and $t_{0.9} = -\tau \ln(1 - 90)$.

The rise time t_R refers to the time it takes for the leading edge of a pulse to rise from its minimum $t_{0.1}$ to its maximum value $t_{0.9}$ expressed by

$$t_R = t_{0.9} - t_{0.1} = 2.197\tau. \quad (\text{A.13})$$

Conversely, the fall time t_F is the measure of the time it takes for the pulse to move from the highest value to the lowest value. Other important parameter is the 3dB bandwidth, which is a measure of the range of electrical frequencies a system supports,

$$f_R = \frac{0.35}{t_R}. \quad (\text{A.14})$$

Rise time and 3 dB bandwidth are two closely-related parameters used to describe the limit of a system's ability to respond to abrupt changes in an input signal. They are very convenient, as they do not require measurement of the input to the first order system, but there is no straightforward measure for estimation errors.

On the other hand, classical estimators of linear time-invariant systems are readily applicable when measurements of input and output variables are available, namely the

Frequency Response Function (FRF) estimators. The FRF is an essential representation in dynamic systems that accurately describes the amplitude and phase of the system's response concerning the excitation frequency (CRAIG ROY R.; KURDILA, 2006). It provides critical insights into a system's ability to respond to different frequencies and is an indispensable tool in the analysis, modeling, and control of dynamic systems. FRF estimators play a pivotal role in the thorough characterization of linear time-invariant systems, providing a solid foundation for the study of these systems in the frequency domain. One of such estimators is defined in frequency as (SHIN; HAMMOND, 2008)

$$H_1(\omega) = \frac{S_{xy}(\omega)}{S_{xx}(\omega)}, \quad (\text{A.15})$$

where $S_{xy}(\omega)$ and $S_{xx}(\omega)$ are the cross-PSD (CPSD) between the input x and output y and auto-PSD of the input x .

The ordinary coherence function γ measures the linear relation between an input signal and an output signal in a system, in the frequency domain. It ranges from 0 to 1, indicating the degree of linear relation between the signals (BENDAT; PIERSOL, 2012). The coherence function is equal to 1 in the case of $x(t)$ and $y(t)$ being linearly related. Otherwise, the function equals zero if the two signals are uncorrelated. If the coherence function is greater than zero but less than one, then $x(t)$ and $y(t)$ are partially linearly related. The ordinary coherence function γ is given by the following equation,

$$\gamma(\omega) = \frac{|S_{xy}|^2}{S_{xx}(\omega)S_{yy}(\omega)}. \quad (\text{A.16})$$

Coherence is useful for assessing the quality of experimental measurements and identifying the presence of nonlinearities or noise in the data. It provides information about the reliability of the relation between input and output in dynamic systems. Possible reasons for the departure from a linear relationship between the signals include the presence of noise in the measurement, $x(t)$ and $y(t)$ may not be linearly related (presence of nonlinearities), and $y(t)$ is an output affected not only by input $x(t)$ but also by other inputs.

A.7 Experimental setup

In this section, the experimental setup for the dynamic characterisation of the resistive sensor is presented. The proposed setup aims at producing a step input on the sensing element from a single-phase controlled water inlet, which is tracked with a high-speed camera. In addition, a second experimental bench producing different patterns of two-phase flow for the application of the sensor array is also briefly presented. The results presented in this work were obtained from experimental tests carried out at the

A.7.1 Dynamic characterisation setup

The experimental test of characterisation of the sensors consists of fixing the resistive sensor vertically, as shown in Figure A.7 and applying a single-phase controlled water inlet. The system input is acquired through filming with a high-speed camera where an image processing algorithm is applied in order to capture the interface, that is, the real step input. The high-speed camera used in the experimental test was a Phantom model VEO 640, with a rate of 800 fps and a resolution of 1600×2560 . In this way, both the input and the output response of the resistive sensors are captured. In addition, the time series were stored at an acquisition rate of 800 Hz for 600 seconds using a National Instruments (NI) acquisition system.

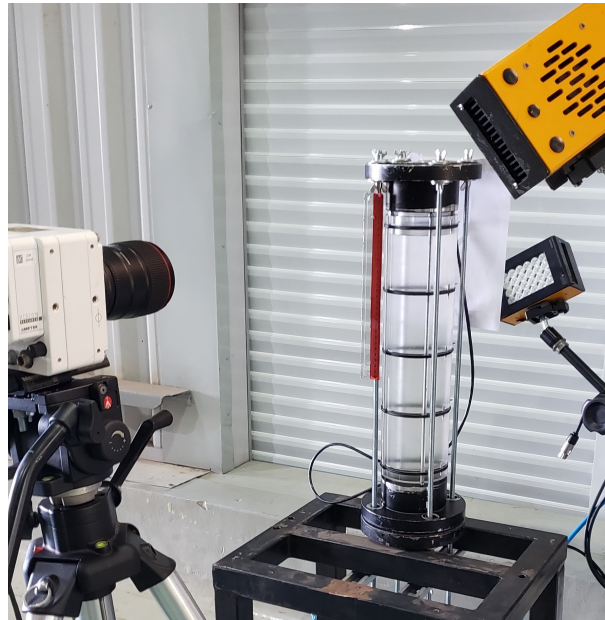


Figura A.7 – Experimental setup for dynamic characterisation with the sensor array with the water inlet, high-speed camera positioned parallel to the resistive sensor in the vertical and lightning.

A.7.2 Two-phase flow bench

Experimental investigations were carried out on a three-inch pipeline system employing the previously mentioned void fraction sensor. A horizontal two-phase liquid-gas flow was established, encompassing a 6-meter section of steel pipe with 4.5 mm thickness. Prior to the measurement section, there is a 20-meter pipeline length for flow development purposes. The working fluids consisted of tap water and compressed air, with properties as follows: liquid density, liquid viscosity, gas viscosity, and surface tension were equal to 998 kg/m^3 , 1 cP , 0.001 cP , and 0.07 N.m , respectively. The gas density

was adjusted according to line pressure and temperature at the measurement section. Figure A.8 illustrates one module comprising four resistive sensors (R_1 , R_2 , R_3 , and R_4) for void fraction measurement. Furthermore, time series data were recorded at an acquisition rate of 25.6 kHz using a National Instruments (NI) data acquisition system. A transparent plexiglass visualisation section was integrated into a segment of the test bench housing a high-speed camera (Phantom model VEO 640). The primary purpose of employing the camera was to visually discern the flow pattern. Notably, measurements from all other sensors fall beyond the scope of this study.

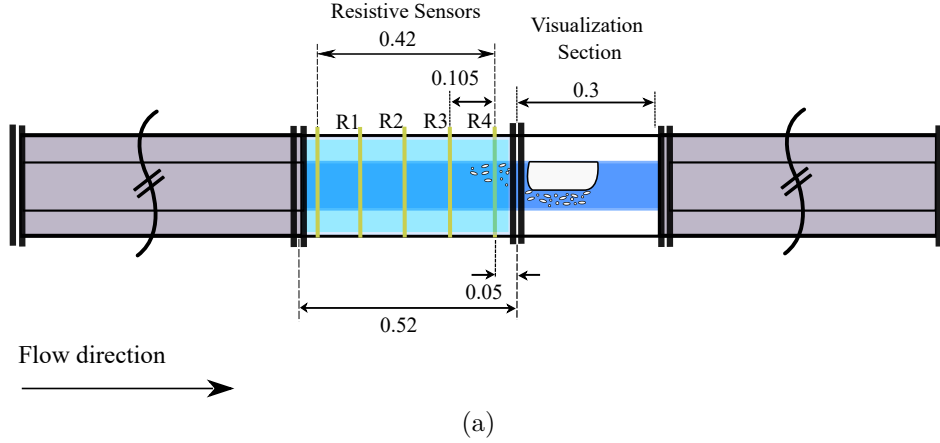


Figure A.8 – Two-phase test bench: (a) schematic with distances between resistive sensors, and (b) a photo of the experimental setup highlighting the visualisation section and the void fraction sensor.

Flow patterns describe how the liquid and gas phases interact and geometrically distribute within a conduit or pipe. However, there is a difficulty in defining two-phase flow patterns and confirming them is related to the complexity of the interactions between the two fluids and changes in flow conditions, such as velocity, pressure, and phase proportion. For this reason, in this work, the direct visualisation method using the high-velocity camera and direct measurement using resistive sensors are used to confirm the flow patterns. In experimental tests, with the configuration and mixtures detailed previously, it was possible to observe four flow patterns. The table 13 shows the 23 experimental points acquired in the experimental test, where the liquid surface velocities (J_{sl}), gas surface velocities

(J_{sg}), mixture velocities (J_m) and the flow pattern are detailed. The flow pattern is created given these flow conditions and a sufficiently long development length for the flow. The flow pattern was defined by observing the images from the high-velocity camera and the signals from the resistive sensors, which will be detailed in the next section.

Tabela 13 – Experimental test matrix with liquid surface velocities (J_{sl}), gas surface velocities (J_{sg}), mixture velocities (J_m) and flow pattern observed by high-velocity camera. The flow patterns are Smooth-Stratified (SS), Stratified-Wavy (SW), Slug (SL), and Dispersed Bubble (DB).

Point	J_{sl} , m/s	J_{sg} , m/s	J_m , m/s	Flow pattern
1	0.258	0.106	0.364	SS
2	0.259	0.203	0.461	SS
3	0.258	0.481	0.738	SS
4	0.258	0.899	1.157	SW
5	0.258	1.145	1.402	SW
6	0.258	1.822	2.079	SW
7	0.259	2.036	2.295	SW
8	0.580	0.504	1.083	SL
9	0.579	0.977	1.556	SL
10	0.578	1.429	2.008	SL
11	1.191	0.382	1.572	SL
12	1.315	0.845	2.159	SL
13	1.353	1.400	2.756	SL
14	1.300	0.493	1.793	SL
15	1.242	1.001	2.243	SL
16	1.212	1.218	2.429	SL
17	2.150	0.532	2.681	SL
18	2.044	1.017	3.060	SL
19	2.017	1.182	3.199	SL
20	3.153	0.095	3.248	DB
21	3.042	0.421	3.462	SL
22	2.927	0.854	3.782	SL
23	3.328	0.078	3.406	DB

A.8 The cross-talking issue

In this section, the experimental results from the switching and non-switching sensor configurations obtained at the bench for dynamic characterisation, Section A.7.1, are presented and discussed. It is shown that cross-talking between subsequent sensors is an issue at the non-switching configuration of the sensor array. This is highlighted due to controlled input generated by a single-phase water inlet. Subsequently, experimental results from the switching sensor configuration are presented and discussed. In both cases, the corresponding demodulation schemes are also presented and discussed.

A.8.1 Non-switching configuration

In the first configuration, the sensor array is excited with a carrier sine wave with a central frequency of 1.6 kHz in the non-switched configuration, aimed at assessing the impact of parasitic impedances. As depicted in Figure A.9(a), both the temporal and spectral profiles of the carrier wave are presented. The output signals, in the time and frequency domains, are displayed in Figure A.9(b), respectively. The output signal has the same frequency as the input signal but its amplitude is modulated in time from close to 0 V to approximately 2.5 V. During the measurement time, the sensor array is filled with a controlled water inlet and the output amplitude jumps at the time the water level hits the sensing element. Then, the water level is decreased again until that there is no more water in the sensing element and the output amplitude drops to close to zero again. The process is repeated once more during the measurement time. The resulting signal is an amplitude-modulated waveform that carries the information about the liquid height in the pipe.

In the demodulation process, the carrier wave is suppressed, allowing for the recovery of the original information at the output. This procedure is carried out in the post-processing phase of the signals and essentially involves the application of a zero-phase Butterworth low-pass filter to the output signal, tuned to the central frequency of the carrier wave, followed by the extraction of the signal envelope, as outlined in Section A.4. Figure A.9 (b) shows the temporal signals of the resistive sensors in the experiment, with the demodulation results represented in red line.

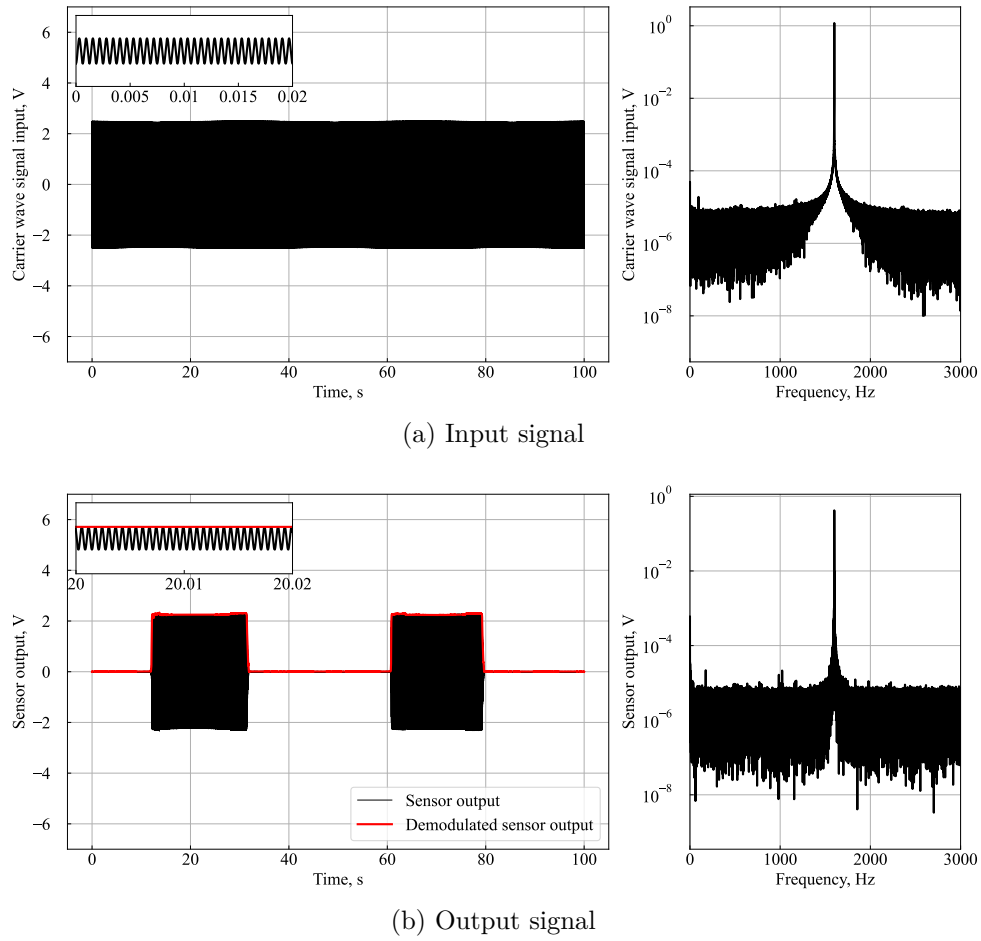


Figura A.9 – Sinusoidal carrier (a) and output signal or sensor response (c) and their respective frequency response (b) and (d) Sensor response (blue lines) of the resistive sensors and the demodulated signals (red lines).

After the demodulation process, the output signal from the sensor is calibrated using full and empty references, Section A.5. Figure A.10(a) shows the calibrated sensor output with two filling and emptying cycles. Given the vertical sensor array configuration, the response sequence of the sensors is R_4 , R_3 , R_2 , and R_1 . Consequently, R_1 is the first to respond to the arrival of the unitary pulse generated by the experiment and remains in a high state for a longer duration as the pulse propagates towards the other probes. From Figure A.10(b), it is possible to see that there is a clear change in the amplitude of the void fraction every time a subsequent sensing element is crossed by the water level. These fluctuations are attributed to inherent parasitic impedances within the measurement system, leading to undesired interaction between the sensors, i.e., cross-talking. At around 9 s of measurement, the response from sensor R_2 is directly influenced by the rapid change in impedance at sensor R_3 . There is an increase in amplitude for R_2 while there is a decrease in amplitude for sensor R_3 . Similarly, just after 13 s, the abrupt change in sensor R_3 directly influences the response of sensor R_2 . Notice that this effect is most evident during the changes of water levels, which highlights its dynamic characteristic. These amplitude variations are a common challenge in measurement systems due to the presence of reactive components and coupling among the sensors, which requires additional actions

to mitigate and obtain more accurate void fraction measurements when using an sensor array.

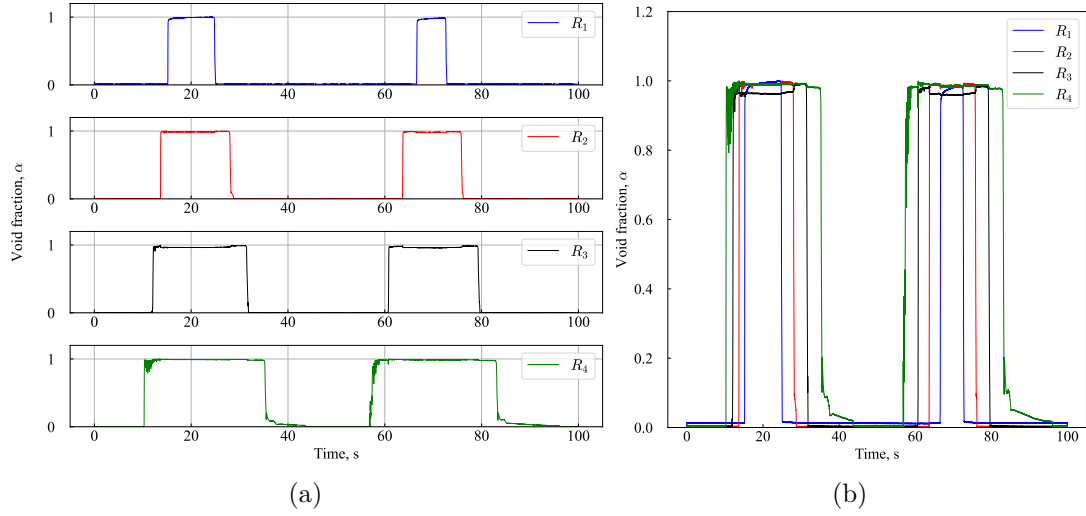


Figura A.10 – Response of resistive sensors using the non-switching configuration in terms of void fraction (a) and a highlight in a unit cell showing the effect of cross-talking (b)

Notably, this result suggests that a similar phenomena occurs when the sensor is applied in two-phase fluid transport systems, which is obscured in the typical experimental conditions due to the dynamic features of the fluid flow.

A.8.2 Switching configuration

In the switching system configuration, as illustrated in Figure A.5, a single signal output is obtained. The sensor array is excited by a sinusoidal carrier wave $q(t)$, characterised by a central frequency of $f_q = 3200$ Hz, a sampling rate of $f_s = 25.6$ kHz, and a switching frequency of $f_{CH} = 400$ Hz. Figure A.11 presents the input and the output of the resistive sensors in the time and frequency domains. Furthermore, the sensor's output in the time domain exhibits a more complex profile resulting from the mixing of information from the four sensor elements. Additionally, it can be observed that the output signal has a richer spectrum attributed to the system switching, which introduces significant harmonic components.

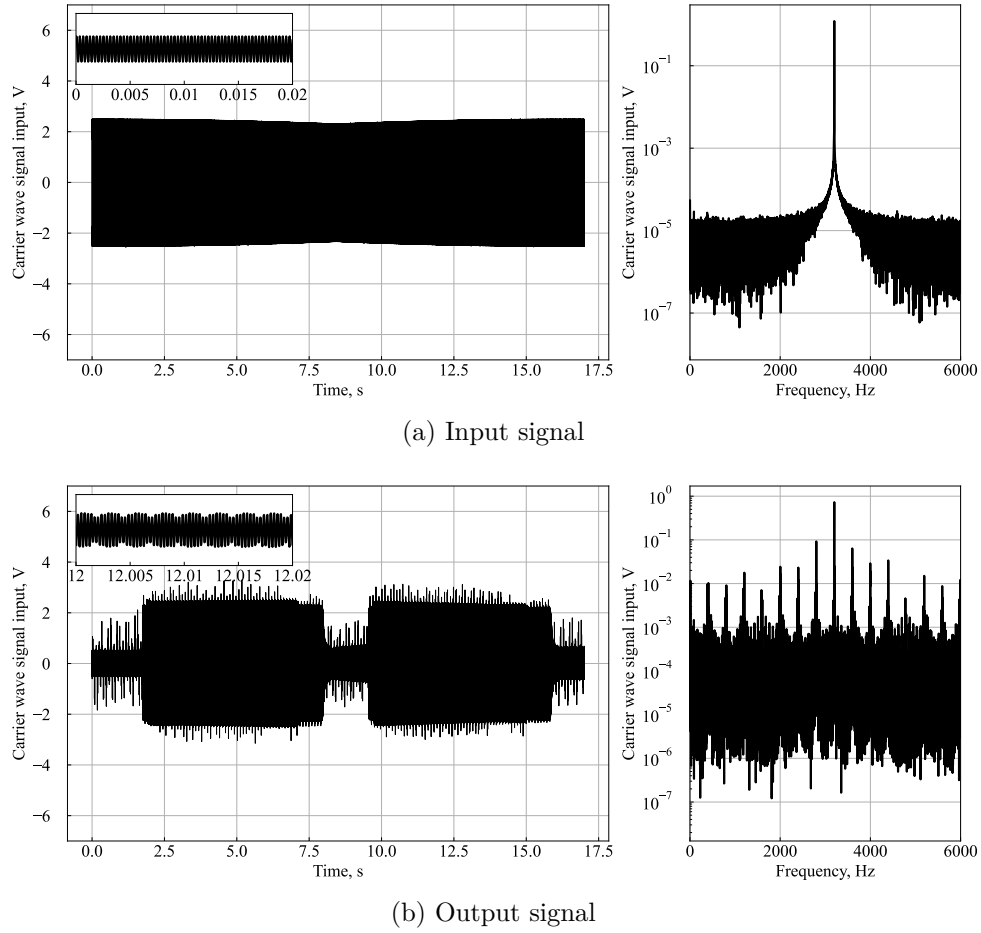


Figura A.11 – Input signal from the resistive sensor or sine carrier wave (a) and its respective DFT (c) and output signal from the sensors (b) and its respective DFT (d).

Figure A.12 shows both digital outputs D2 and D3 that controls the switching sequence, the signal at each resistive sensor R_1 , R_2 , R_3 and R_4 , and the corresponding activation pulse train AT_{R1} , AT_{R2} , AT_{R3} and AT_{R4} for each sensor, resulting from the synchronous combination of D2 and D3, as shown in Table 12. Notice that, where each active segment has a sine wave coming with amplitude and phase information about the sensing element with respect to the input carrier wave. Moreover, each activation pulse will result in a single value from Equation A.6, as discussed in Section A.4.2.

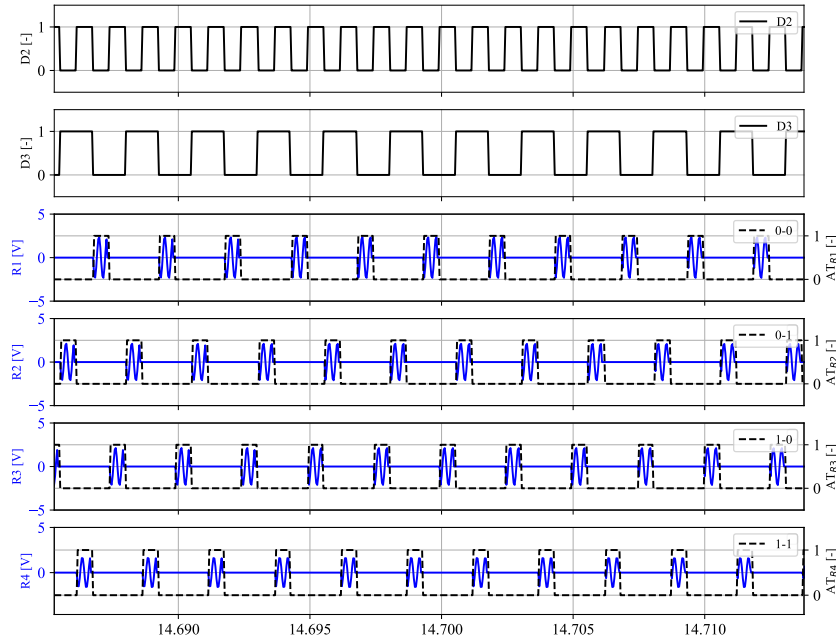


Figura A.12 – Micro controller outputs (D2 and D3) and activation signals of each sensor (AT_{R1} , AT_{R2} , AT_{R3} and AT_{R4}) based on the table.

Figure A.13 presents the sensor output and the corresponding demodulated signal for sensors R_1 , R_2 , R_3 and R_4 after the demultiplexation along with the corresponding demodulated signal. A similar behaviour is observed when compared to non-switching configuration, as expected. Similar to the non-switching approach, a linear model is assumed between the voltage output of the sensor and the liquid height. The gain $|G_i^j|$ is estimated from Eq. A.6 for the i^{th} sensor at the j^{th} time sample. The calibration procedure is subsequently used for calibrating the sensor array, such that the mean value and variance of the liquid height is the same at every sensor in the array. Finally, similar to the non-switching case, the non-linear relation between the liquid height and the void fraction is applied. Figure A.13(a) presents the void fraction measurement for several cycles of filling up and draining out the sensor array with single-phase water. Figure A.13(b) highlights one of the cycles. It is possible to note the switching approach successfully avoided cross-talking between the sensors output in the array, as there is no clear influence from the output between sensors during the abrupt changes of void fraction values.

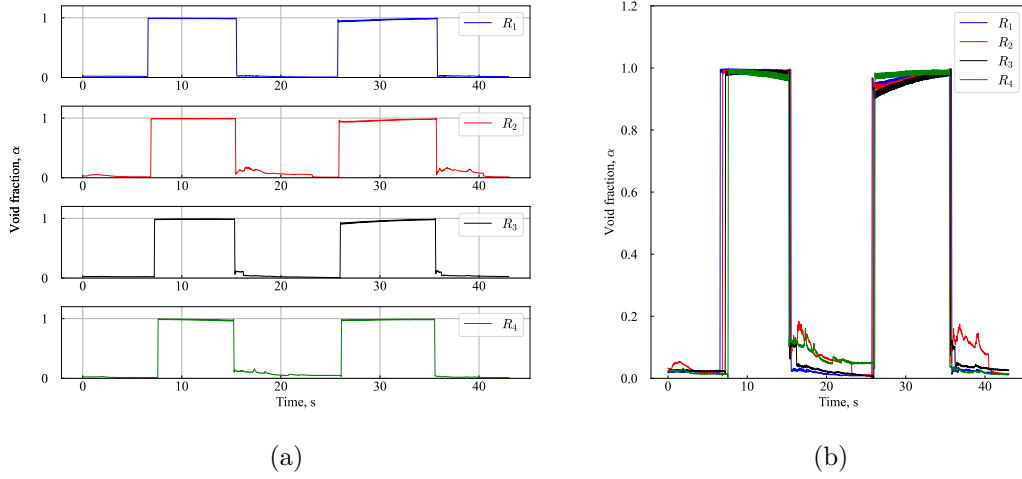


Figure A.13 – Response of resistive sensors using the switching configuration in terms of void fraction (a) and a highlight in a unit cell showing the effect of cross-talking (b).

A.9 Experimental estimation of the dynamic response

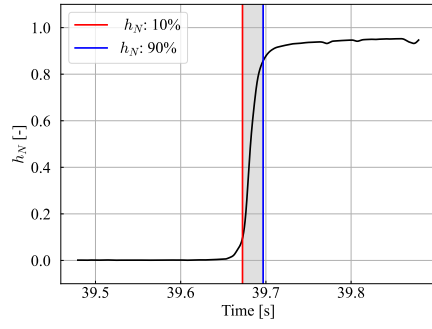
This section investigates the experimental results derived from the dynamic characterization setup with a focus on analyzing the time response, frequency range, and linearity of the phase, as detailed in Section A.7.1. These parameters are of utmost importance in the context of engineering applications where the dynamics of two-phase flow are described in terms of void fraction measurements. The sensor's output is determined by employing the methodologies proposed in the preceding sections. The input is obtained through high-velocity camera measurements. An image processing technique is employed to track the interface between the water column and air, thereby generating a time series of liquid height. This process is synchronized with the liquid height output of the sensor through an analog camera trigger.

A.9.1 Response time and frequency range

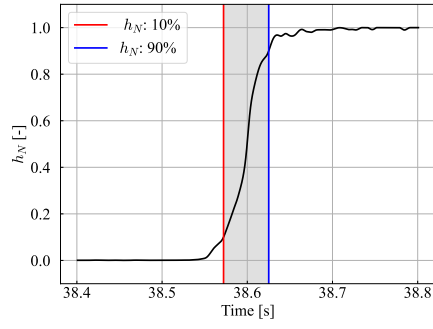
The rise time t_R is a critical parameter in the analysis of systems and sensors. It is a crucial measure to understand how quickly a sensor or system responds to a sudden change in its state. The shorter the rise time, the faster the sensor can detect and respond to changes in its environment. Therefore, rise time is calculated by subtracting the time at which the sensor reaches 10% of its final value from the time at which it reaches 90% of its final value. The resulting value provides us with a measure of how rapidly the sensor responds to the transition from 0 to 1. Another important parameter for understanding the sensor's performance regarding dynamic events is the sensor's frequency response f_R . This has an inverse relationship with the rise time, $f_R = 0.35/t_R$. The wider the frequency

response and the shorter the t_R , the more effective the sensor will be in detecting and responding to rapid changes.

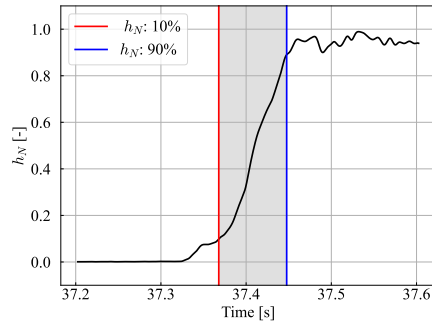
In order to estimate the time and frequency response of the conductive sensors in the array, experiments were carried out with the sensor in a vertical position, involving various liquid rise velocities v_R from the controlled water inlet. The rise velocity was manually controlled through a valve at the sensor's inlet and calculated using two nearby stations. Figures A.14 and A.15 shows the h_N time response for the four sensors in the array, both in switched and non-switched configurations. The signals are filtered by a Butterworth low-pass filter to the output signal tuned to 10 Hz for smoothing. It should be noted that the time axis is modified to highlight the rise time of the sensors. Note that the regions corresponding to 10% and 90% of the final amplitude value have been highlighted in grey. Finally, Table 14 provides a summary of three repetitions with different rise velocities in which t_R and f_R were calculated. It is worth noting that the sensor with switching configuration exhibits shorter time and frequency responses when compared to the non-switched configuration. In this context, the switching configuration introduces delays, resulting in longer response times. This situation was expected, given that switching occurs between four sensors, meaning that three-thirds of the time the sensor is not reading the system. On the other hand, the non-switching configuration, in which all sensors are active simultaneously, can operate more effectively and directly in terms of time and frequency response, resulting in shorter response times. However, as previously analysed, there is cross-talking in the measurements, affecting void fraction estimates.



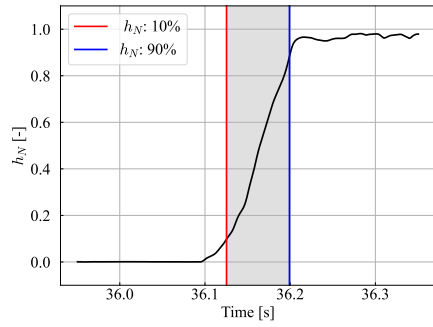
(a) Sensor R1



(b) Sensor R2



(c) Sensor R3



(d) Sensor R4

Figura A.14 – Unit-step input liquid height using the non-switched configuration for each sensor. Highlighted region corresponds to the response time between 10% and 90% limited by the red and blue lines, respectively.

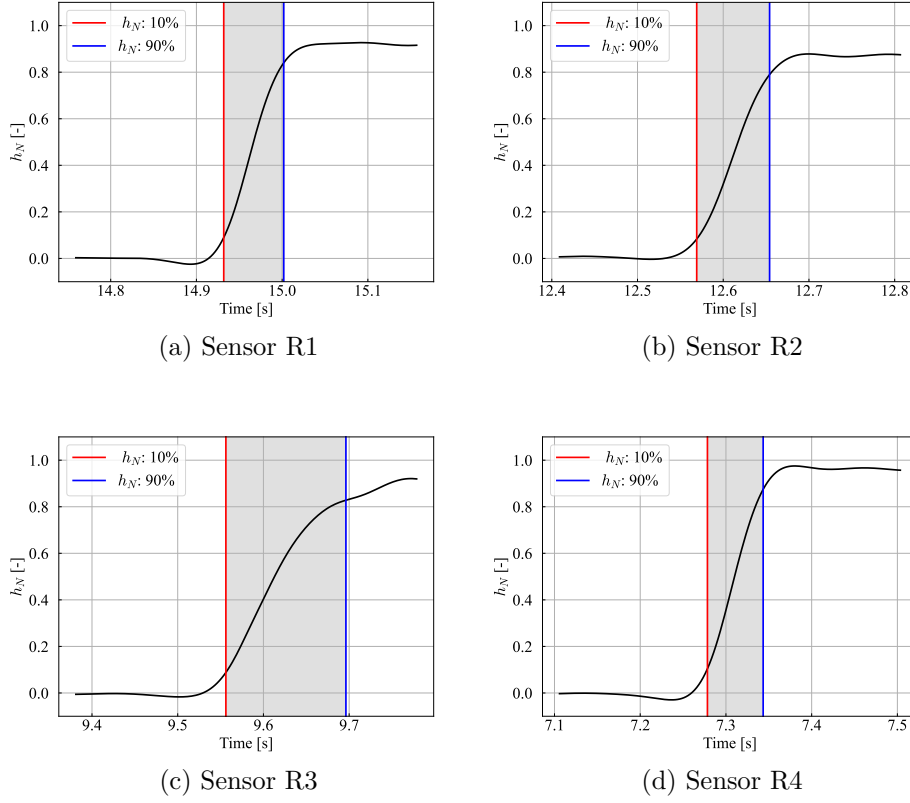


Figura A.15 – Unit-step input liquid height using the switched configuration for each sensor. Highlighted region corresponds to the response time between 10% and 90% limited by the red and blue lines, respectively.

Tabela 14 – Estimated rise time (t_R), rise frequency (f_R) and rise velocity (v_R) at sensors R_1 , R_2 , R_3 and R_4 in the array for the non-switching and switching configurations.

	Non-switching			Switching		
Sensor	v_R [m/s]	t_R [s]	f_R [Hz]	v_R [m/s]	t_R [s]	f_R [Hz]
R1	0.097	0.051	6.889	0.040	0.159	2.197
R2	0.097	0.115	3.039	0.040	0.275	1.274
R3	0.097	0.171	2.051	0.040	0.170	2.055
R4	0.097	0.161	2.175	0.040	0.324	1.080
R1	0.138	0.081	4.343	0.045	0.154	2.276
R2	0.138	0.186	1.879	0.045	0.187	1.874
R3	0.138	0.160	2.184	0.045	0.308	1.138
R4	0.138	0.219	1.595	0.045	0.143	2.451
R1	0.060	0.109	3.199	0.182	0.143	2.450
R2	0.060	0.219	1.600	0.182	0.159	2.197
R3	0.060	0.071	4.937	0.182	0.198	1.770
R4	0.060	0.121	2.901	0.182	0.143	2.450

Furthermore, it was observed that the time and frequency response change with the liquid rise velocity. This is expected since sensors with thermal masses such as impedance

and capacitance exhibit inertia and take some time to respond to changes in conditions. In summary, estimating the time and frequency response of sensors is crucial in applications for monitoring multiphase systems where rapid and abrupt phase changes occur. The broader the frequency response and the shorter the rise time, the more effective the sensor will be in detecting and responding to rapid changes in its environment.

A.9.2 Liquid height tracking from high-velocity camera

During the characterisation test, the sensor array was positioned vertically, resulting in the synchronised acquisition of time series data from the sensor outputs and images of the liquid height obtained through a high-speed camera. This process resulted in a Single Input, Single Output (SISO) system with inputs and outputs. The images were recorded at a 2000 frames per second (fps) with a resolution of 1536×1440 , and a millimetre-scale ruler was used for spatial reference and subsequent velocity calculation, as can be observed in the Figure A.16 (a).

To detect the liquid interface, it was necessary to perform an image processing task, which included the noise reduction procedure by the Gaussian blur function of the 3×3 kernel. After that, the absolute difference between the two images was calculated with a step of 30 frames, it can be seen that it was possible to detect part of the interface in motion. Then, a binarisation based on the Otsu method (OTSU, 1979) was applied, returning a single intensity threshold that separates the pixels into two classes, foreground, and background. Finally, morphological operations were applied on the binarized image. First the image was dilated by a 1×15 kernel, then the erosion operation was applied with a 2×2 kernel to smooth the blank interface. At the end of this operation, it is possible to fully detect the interface as shown in Figure A.16 (b) for a single frame.

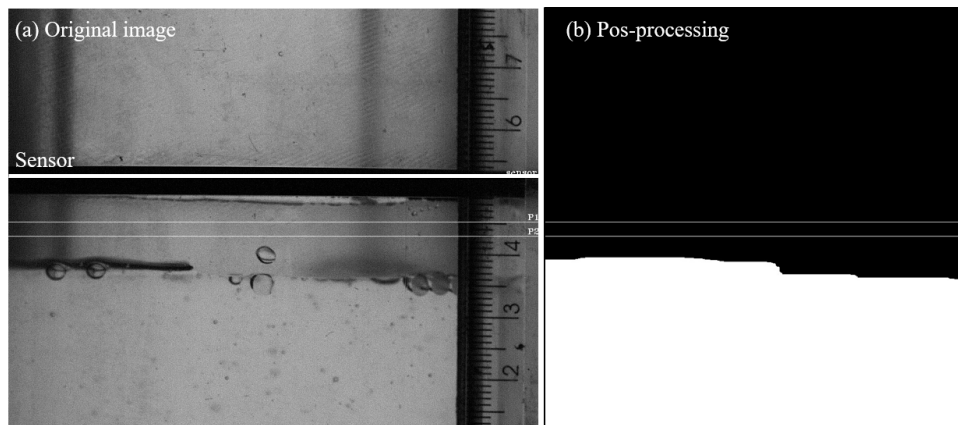


Figura A.16 – Original image (a) and the result of image processing (b).

Figure A.17 B displays the liquid height profiles, both from the sensor response and the image processing. It is noticeable that the curve associated with the image exhibits a quicker response, reaching the high level simultaneously with the liquid height increase

when compared to the sensor response. In addition, it can be seen that the water level has a spatial profile due to the flow features. Thus, it gradually touches the wires from the sensing element as the water level rises, following this profile, which reduces the step frequency bandwidth, as discussed in the following section.

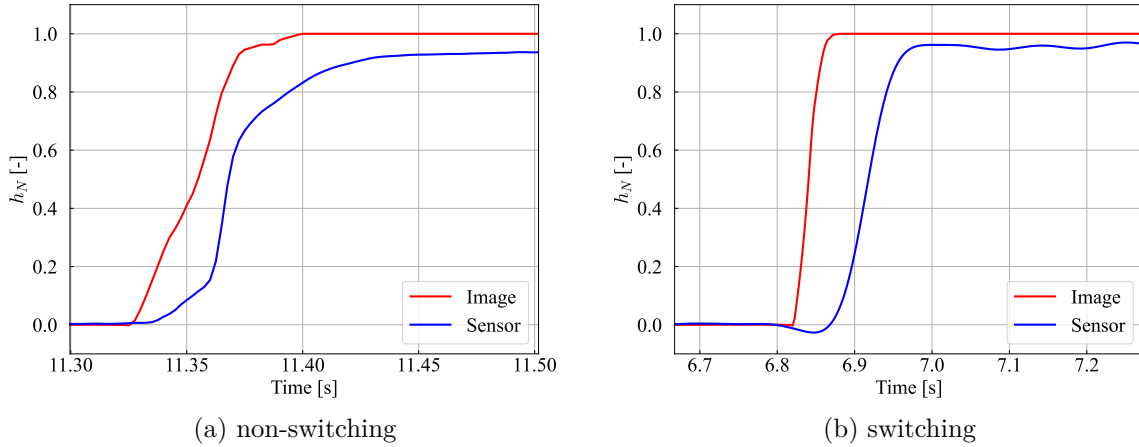


Figure A.17 – Liquid height estimated from the image processing (blue line) and from the sensor (red line) for the (a) non-switching and (b) switching configuration. Position of the sensor is highlighted by a white line.

A.9.3 Frequency response estimation

In this section, the frequency response function of the sensing element plus the corresponding electronic circuits is estimated from the input response using the H_1 estimation, as briefly presented in Section A.6. The cross-PSD and auto-PSD of the sensor input and output are estimated using the Welch's approach (SHIN; HAMMOND, 2008; BENDAT; PIERSOL, 2012). Both switching and non-switching sensor configurations are used with three independent time measurements for each case. Figure A.18 shows the estimated auto-PSDs for the sensor input, measured from the high-speed camera, and the output, measured from the sensor. Notice that most of the PSD energy is concentrated up to about 2 Hz, in the non-switching test, and up to about 1.5 Hz, in the switching test. This limitation in frequency band is directly linked to the limits of the test bench, presented in Section A.7.1. Increasing this frequency band would require increasing the rise time of the liquid interface. However, this would also yield a non-flat interface, which also limits the frequency band. A systematic investigation of the optimal setup is beyond the scope of this paper and is subject to further investigation.

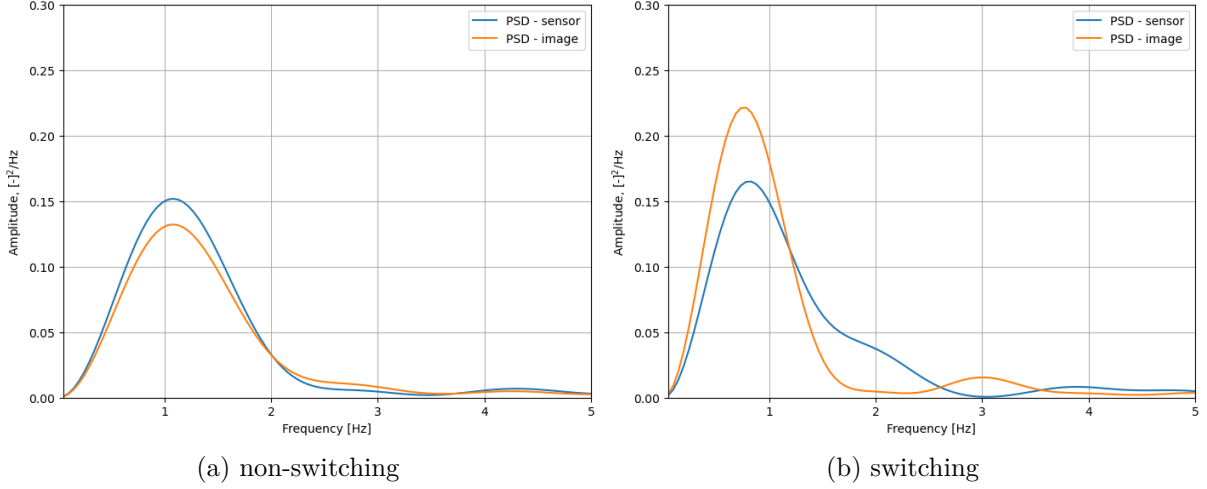


Figure A.18 – Estimated auto Power Spectral Density (PSD) from the image processing (input) and sensor (output) for the (a) non-switching and (b) switching configurations.

Figure A.19 shows the estimated amplitude and phase of the Frequency Response Function (FRF) from the H_1 estimator and also the corresponding coherence function for the non-switching and the switching configurations. Notice that for both cases, the frequency band with coherence closer to unity are those in which the input and output PSDs, Figure A.18, have higher energy. This highlights the limitation of the experimental setup rather than the sensor itself. Most importantly, this frequency band presents a constant gain close to unity and linear phase. The constant gain assures that only a static constant is necessary to calibrate the sensor output. For both cases, this gain is not exactly one because of the off-set calibration of the output, as shown in Figure A.17. The linear phase region indicates that the sensor responds with no distortion, i.e., a simple delay. In this case, the complex-valued FRF can be given as $H(\omega) = |H(\omega)| \exp(-j\omega\Delta t)$, where Δt is the time delay. Figure A.19 also presents a linear mean-square error fit in the linear phase band, with the corresponding Δt estimated. Notice that the switching configuration presents almost twice as much delay as the non-switching, which is consistent with the estimated rise time for each case.

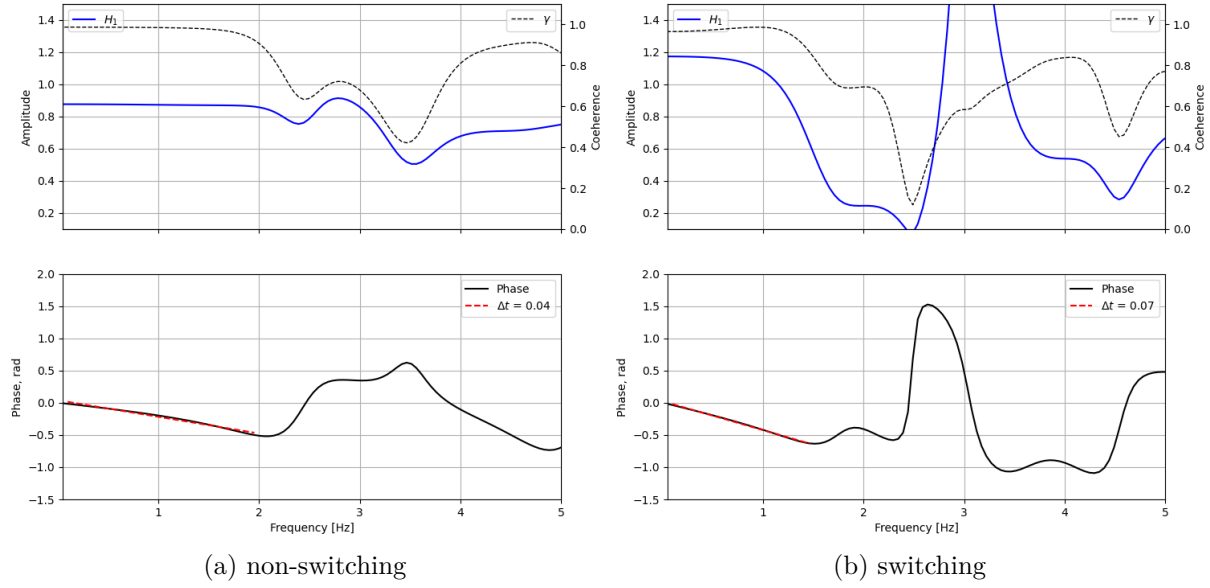


Figure A.19 – Estimated frequency response amplitude and coherence (upper) and phase (lower) for the (a) non-switching and (b) switching configurations. Estimated linear phase delay (dashed red) and corresponding delay time Δt .

A.10 Validation

In this section, the demodulation methodology from the switched configuration is applied to the void fraction data obtained in the experiment in the setup detailed in Section A.7.2. Figure A.20 presents 15 seconds of a typical void fraction measurement from the sensor array at four experimental points: stratified, wavy-stratified, slug, and dispersed bubbles. In the stratified pattern, the void fraction signal remains relatively constant over time since the liquid and gas phases are well-separated in distinct layers. In the wavy-stratified pattern, similar to the stratified flow but with some wavy variations due to flow disturbances, the void fraction signal may exhibit minor oscillations but will still remain relatively constant on average. However, the signal in the slug pattern is transient, characterised by the presence of a liquid slug and Taylor bubble (VIEIRA et al., 2023). As for the dispersed bubbles pattern, the void fraction signal is continuous and close to zero. Table 15 summarises the mean value and standard-deviation from each sensor at each experimental point. Notice that they are all identical, as constrained by Equation A.8. As a result, the validation is performed using just one void fraction measurement for each experimental point.

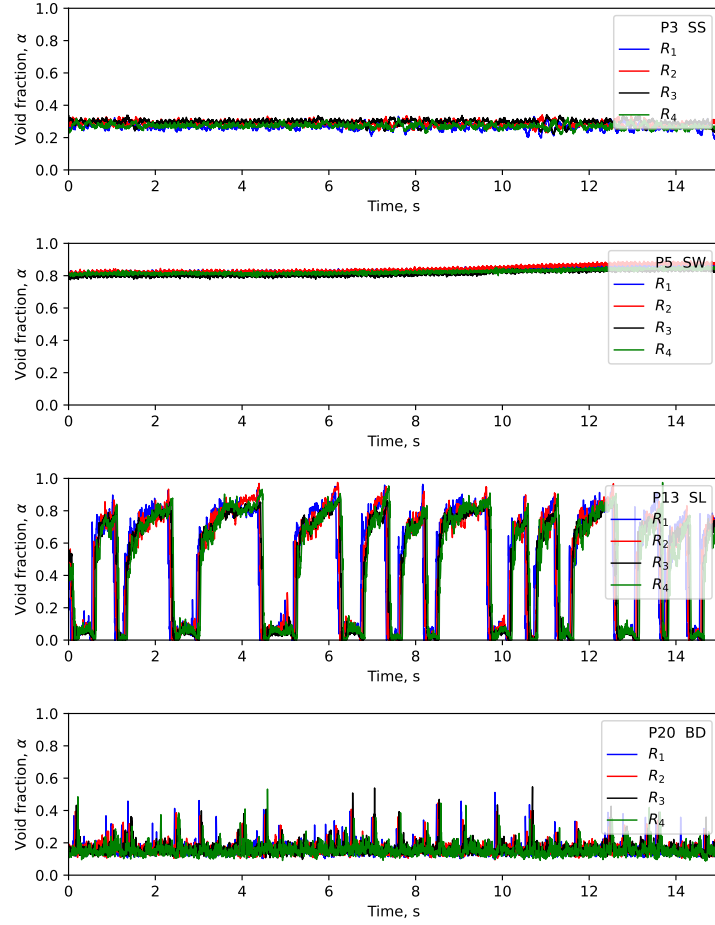


Figura A.20 – Time series of void fraction for the stratified (SS), wavy-stratified (SW), slug (SL), and dispersed bubbles (DB) patterns.

Tabela 15 – Flow pattern, mean value and standard deviation of each experimental point.

Experimental point	mean value, μ				standard-deviation, σ			
	R1	R2	R3	R4	R1	R2	R3	R4
α_1 , stratified	0.08	0.08	0.08	0.08	0.04	0.04	0.04	0.04
α_2 , wavy-stratified	0.69	0.69	0.69	0.69	0.20	0.19	0.19	0.20
α_3 , slug	0.61	0.61	0.61	0.61	0.27	0.27	0.27	0.27
α_4 , dispersed bubbles	0.17	0.17	0.17	0.17	0.02	0.02	0.02	0.02

Figure A.21 shows the Probability Density Function (PDF) of void fraction times, estimated using a kernel density estimator (KDE), for each experimental point in the test matrix, Table 13, reflecting the flow conditions and sensor measurements. It is noteworthy that in the stratified patterns P1, P2, and P3, where the liquid and gas phases are well-separated in distinct layers, the PDFs show a concentration of void fraction values close to a constant, reflecting the stability of this pattern. In the wavy-stratified pattern, similar to stratified but with wavelike variations due to flow disturbances, the PDF may exhibit small oscillations but is expected to maintain a concentration of values around an average level. In the slug flow pattern, characterized by the presence of liquid slugs and Taylor bubbles, the PDF can be more complex, exhibiting two modes: one at a low void fraction

representing the liquid slug and piston, and the other at a high void fraction representing the Taylor bubble. Finally, in the dispersed bubbles pattern, the void fraction is continuous and close to zero, reflecting the predominance of the liquid phase. As proposed by Vieira et al. (VIEIRA et al., 2023), the Otsu method was used to segment the time series in two regions, represented by the red dashed line. Notice that this method is capable of finding a threshold that separates the low void fractions associated with the liquid pistons from the high void fractions related to Taylor bubbles.

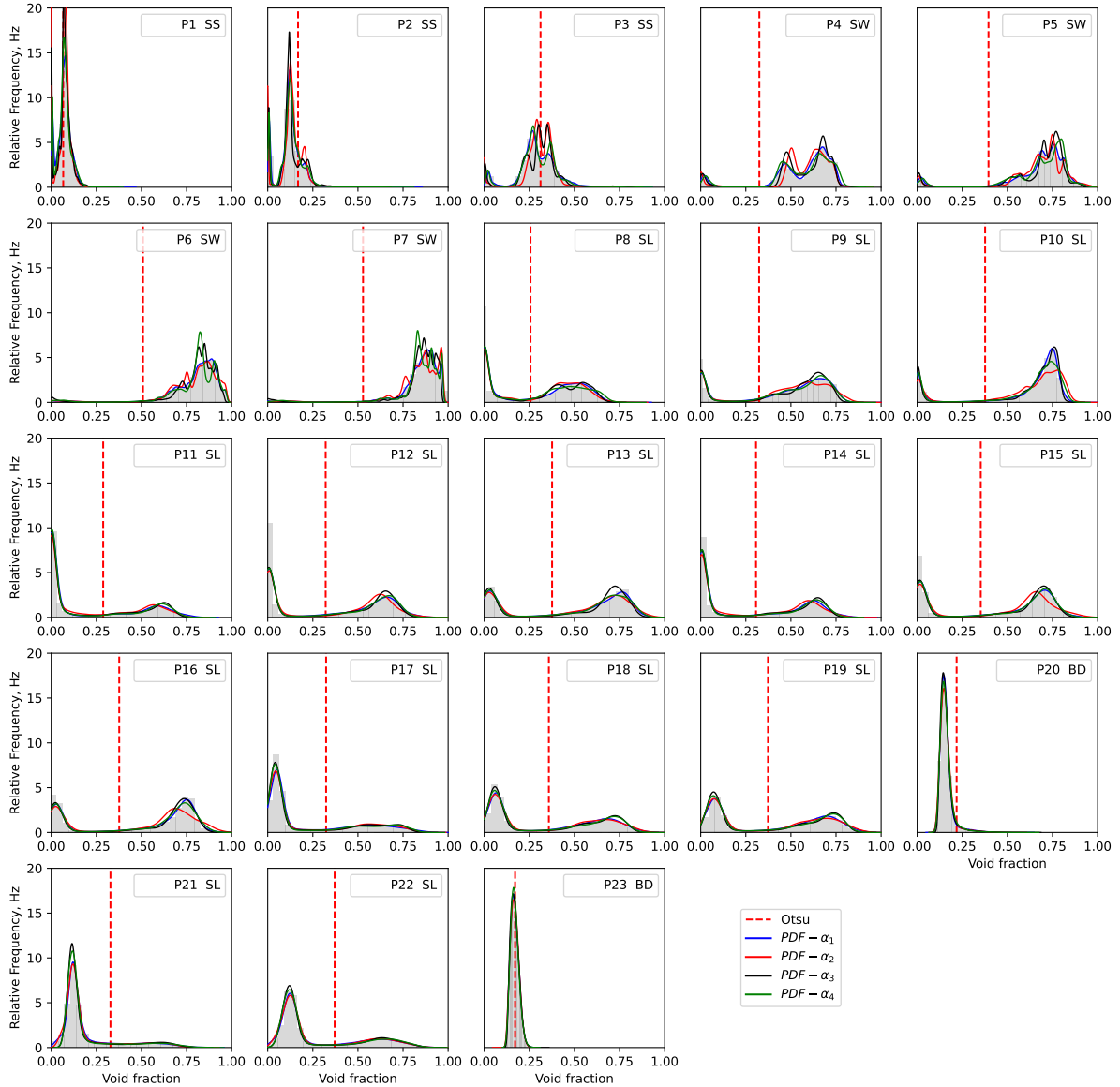


Figure A.21 – Probability Density Function estimated from the void fraction time series for all experimental points of the test matrix, including stratified (SS), wavy-stratified (SW), slug (SL) and dispersed bubbles (DB) patterns. The red dashed line corresponds to the Otsu threshold (red dashed line).

In order to perform the validation of the void fraction measurements, the average void fraction for experimental point is measured using other techniques. For the stratified and stratified-wavy experimental points, the images from the high-velocity camera are used to estimate the level of liquid and then Equation A.12 is used to estimate the mean

Tabela 16 – Average void fraction estimated for each experimental point using the sensor array in switching configuration, the reference technique, the corresponding error and relative error the flow pattern given as stratified (SS), wavy-stratified (SW), slug (SL) or dispersed bubbles (DB).

Point #	Sensor	Reference	Error	Relative error %	Flow Pattern
1	0.09	0.10	0.01	11.1	SS
2	0.20	0.19	-0.01	5.0	SS
3	0.32	0.28	-0.04	12.5	SS
4	0.44	0.51	0.07	15.9	SW
5	0.84	0.75	-0.09	10.7	SW
6	0.91	0.76	-0.15	16.5	SW
7	0.85	0.76	-0.09	10.6	SW
8	0.22	0.37	0.15	68.2	SL
9	0.33	0.58	0.25	75.8	SL
10	0.46	0.62	0.16	34.8	SL
11	0.14	0.17	0.03	21.4	SL
12	0.24	0.31	0.07	29.2	SL
13	0.32	0.42	0.10	31.3	SL
14	0.15	0.15	0.00	0.0	SL
15	0.25	0.33	0.08	32.0	SL
16	0.29	0.41	0.12	41.4	SL
17	0.11	0.12	0.01	9.1	SL
18	0.20	0.25	0.05	25.0	SL
19	0.22	0.29	0.07	31.8	SL
20	0.16	0.03	-0.13	81.3	DB
21	0.08	0.07	-0.01	12.5	SL
22	0.13	0.15	0.02	15.4	SL
23	0.17	0.02	-0.15	88.2	DB

void fraction. This method, however, cannot be easily applied to estimate the void fraction in slug flow, because of its intermittent behaviour. Instead, the quick-closing valve method is used. This method consists in using a pair of valves placed on each side of the test section that are simultaneously closed, trapping the flow on the test section. Afterwards, the liquid is drained to a recipient and weighed. The void fraction is a function of the volume of liquid drained and the volume of the line (GARDENGHI et al., 2020). For the dispersed bubble flow, the homogeneous model is an adequate method to estimate the mean void fraction, once there is no significant slip between the phases, and therefore the mean void fraction can be given by J_{sg}/J_m . The void fraction values obtained using this method are used as reference values to validate the measurements made using the resistive sensors. The values obtained are summarised in Table 16. The maximum absolute error is 0.25, however, the error is usually lower than 0.15. It is interesting to note that relative errors are typically the lowest in the stratified and stratified wavy points and the larger errors are in the dispersed bubble points, which have the void fraction super estimated.

A.11 Conclusion

This paper presented an investigation of the dynamic response of conductance-based void meter array. It was shown that the typically used configuration of the conductance measuring circuit can present cross-talking when two or more sensors are used simultaneously. A new electronic circuit was proposed, which switches between sensors in the array, consequently avoiding the cross-talking. This effect was clearly shown from the step input response of the array, in which the sensor response were drastically affected by impedance change in the adjacent sensor. In order to show it, a bench setup oriented for the dynamical characterisation of the sensing element was proposed. It has a controlled water inlet, that produced a step input in the sensing element, and a high speed camera, for tracking the water level. This configuration allowed for estimating the frequency response amplitude and phase of sensing element plus the circuitry. It was shown that the proposed experimental setup produced band limited step input up to 2 Hz and that the sensor element presented constant gain with linear phase in this band, in both non-switched and switched configurations. Further work includes improving the band limited step input for extending the characterisation of the sensor.

Due to switching configuration of the proposed electronic circuit, a demodulation approach was proposed for extracting the signal information from the sensor array based on the gain between the input and output of the sensor at each switching cycle. Moreover, a robust calibration approach were proposed for the sensor array which was validated from for several experimental points of different flow patterns. The approach is robust in the sense that the sensor array is calibrated such that the measurements at each sensor in the array have the same statistical properties. This is a relevant feature of the array since all signals are delayed measurements of the same propagating wave.

B Image processing

This chapter introduces the application of the SAM (Segment Anything Model) for bubble segmentation in images of two-phase flows. The SAM, a deep learning model, stands out for its versatility in addressing image segmentation tasks. Here, we explore the effectiveness of SAM in the automatic identification of bubbles in gas-liquid flow experiments, utilizing its unique prompt-based approach.

B.1 Segment anything model

The SAM (Segment Anything Model) is a deep learning model developed for image segmentation tasks. It is designed to be highly flexible and capable of adapting to a wide range of segmentation tasks, such as edge detection, object segmentation, and semantic segmentation, among others (KIRILLOV et al., 2023). The model is trained based on a task called prompt-based segmentation, where it receives instructions in the form of prompts to generate valid segmentation masks for images. This approach allows SAM to be used for solving various segmentation tasks, simply by providing the appropriate prompt, as illustrated in Figure B.1 (a). Additionally, SAM is trained with a large volume of automatically collected segmentation masks, making it robust and generalized for different data distributions. SAM's versatility makes it possible to apply it in various practical scenarios, making it a powerful tool for image processing and object segmentation, including bubble detection in two-phase flow images.

The SAM model generation involves an image encoder, a prompt encoder, and a mask decoder, as illustrated in Figure B.1 (b). Furthermore, SAM can handle prompts of various types, such as point, box, mask, and free-form text prompts. The data engine can be of three distinct types: manual-assisted, semi-automatic, and fully automatic. In the manual-assisted mode, SAM assists annotators in the task of annotating masks, following an approach similar to classical interactive segmentation settings. In the semi-automatic mode, SAM can automatically generate masks for a subset of objects, based on suggestions of possible object locations, while annotators focus on annotating the remaining objects, contributing to a broader diversity of masks. In the fully automatic mode, SAM creates a regular grid of foreground points, resulting in numerous high-quality masks per image. The dataset used, known as SA-1B, includes over one billion masks from 11 million licensed

images. SA-1B, collected fully automatically using the last phase of the data engine, as illustrated in Figure B.1 (c) (HE et al., 2023; ZHANG; JIAO, 2023).

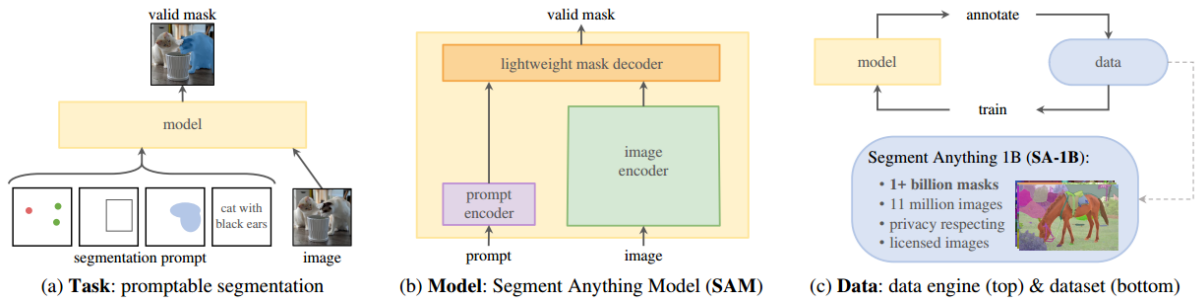


Figura B.1 – Ilustração do método de segmentação de imagem SAM com as etapas de: (a) tarefa de segmentação com prompts, (b) modelo SAM e (c) *data engine* para coletar o SA-1B (KIRILLOV et al., 2023).

The SAM model uses a minimally adapted pre-trained Vision Transformer (ViT) from the Masked Autoencoder (MAE) to process high-resolution inputs. The image encoder runs once per image and can be ViT-B, ViT-L, or ViT-H. These encoders have different parameter counts, with ViT-B consisting of 91 million parameters, ViT-L having 308 million, and ViT-H containing 636 million parameters. According to the literature surveyed, ViT-L offers the best balance between performance and accuracy. While ViT-H is the most accurate, it is also the slowest, and ViT-B is the fastest but sacrifices accuracy. The results presented in this work were processed using the ViT-H encoder (KIRILLOV et al., 2023).

SAM demonstrates broad applicability across various image segmentation tasks, eliminating the previous need for pre-training or fine-tuning. The approach, known as zero-shot, enables performing segmentation tasks on new images without the need for additional data, while maintaining a high level of accuracy. This achievement was made possible through training on over 1 billion real segmentation masks, using a dataset of over 11 million natural images.

B.2 Bubble segmentation using SAM

The SAM method was employed to perform the segmentation of gas bubbles in the images generated by the high-speed camera in the experimental two-phase gas-liquid flow tests. To achieve this, fully automatic mask generation was used, meaning there was no need for any pre-training or manual annotation. The process of automatic mask generation begins with the sampling of input prompts in the form of single points distributed on a grid over the image. For each of these points, SAM is capable of predicting multiple segmentation masks. After this prediction, the masks are filtered based on quality criteria, and any duplicates are removed using a method called "non-maximum suppression". Figures B.2 display the original images (a) and the results of automatic mask generation (b) for

four characteristic regions of the piston-like flow, where each color represents a distinct mask. It is notable that distinct elements, such as the background, bubbles, and interfaces, are segmented by separate masks. Images with objects of different sizes, shapes, and complexities can lead to the generation of varying numbers of masks. Some objects can be represented by a single mask, while others may require multiple masks to capture all the details. For example, the mixing region showed the highest number of masks compared to others. This flexibility of SAM allows it to adapt to a wide range of segmentation scenarios, resulting in different numbers of masks as needed to better represent the objects in the image.

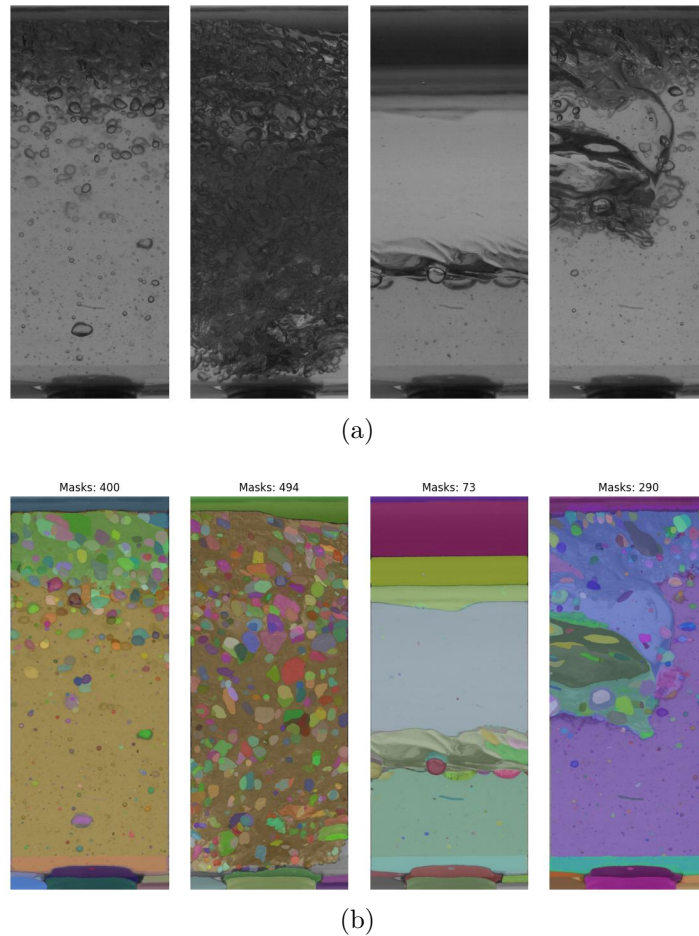


Figura B.2 – Bubble segmentation using SAM for four characteristic regions of the piston-like pattern: Taylor bubble nose, stratified region, mixing region, and liquid piston, from experimental point 13 of the test matrix. The original image (a) extracted from high-speed camera videos and the SAM mask generation result (b) are shown.

The information regarding the bubbles is extracted from the contours identified through mask generation by the SAM model. In this process, an ellipse is fitted to each contour, allowing for the calculation of the equivalent diameter of each bubble. Furthermore, the creation of a bounding box that encompasses the bubble contour, as exemplified in Figure B.3 (b), is of utmost importance for the precise determination of the position and size of each bubble in the image. It is important to note that, in certain situations,

two bounding boxes may arise for the same bubble. This occurs because when SAM generates masks for objects in the image, it can detect different parts of the same object as separate regions. This can result in the creation of multiple overlapping bounding boxes because each mask is based on a different prompt or approach, and these prompts can identify distinct regions of the same object. To mitigate this problem, an approach called Intersection Over Union (IoU) was used, which evaluates the object detection performance by comparing the predicted bounding box to the ground truth bounding box. In this case, it is between the reference box A and its nearest neighbor B (REZATOFIGHI et al., 2019). The formula for calculating the IoU between two overlapping boxes is given by,

$$IoU(A, B) = \frac{(A \cap B)}{(A \cup B)}.$$

The bounding boxes and, consequently, the bubbles detected with an IoU value greater than 0.5 were removed, retaining only those with the largest area. It is important to note that the number of bubbles decreased after this adjustment process, as shown in Figure B.3 (b).

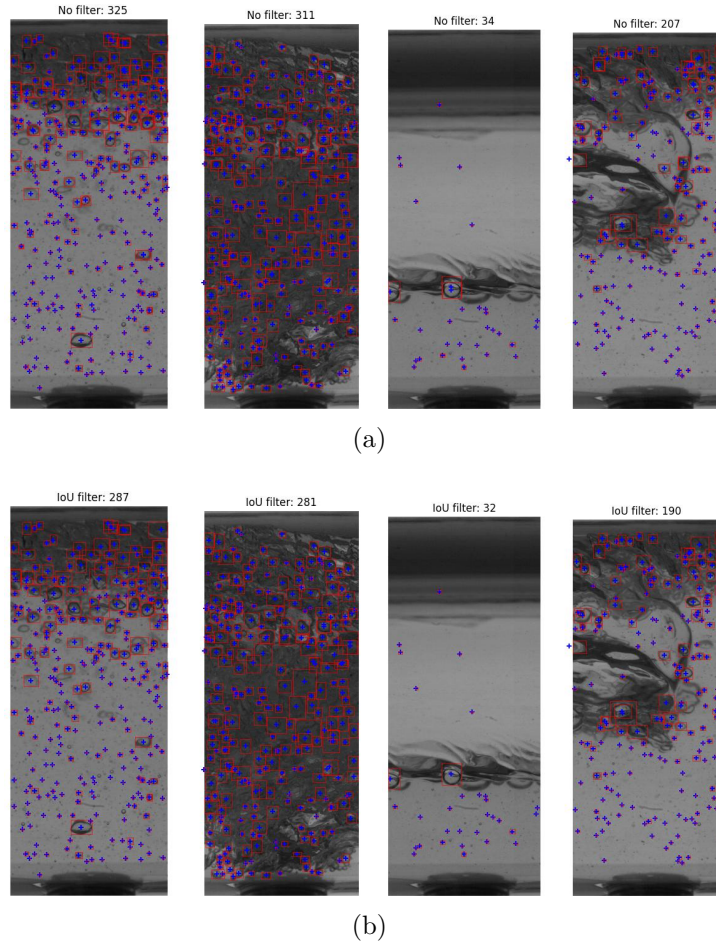


Figure B.3 – Bubble segmentation using SAM for four characteristic regions of the Taylor bubble pattern: Taylor bubble nose, stratified region, mixing region, and liquid slug, from experimental point 13 of the test matrix. The results of bubble detection are shown with each bubble (a) identified by red squares and the bubble filtering using IoU (b).

The bubble segmentation method was applied to all intermittent points listed in Table 3, emphasizing that the videos have approximately 15 seconds in duration at a rate of 800 frames per second. The results related to the number of bubbles and the average diameter throughout the total duration of the video are illustrated in Figures B.4 and B.5. It is worth noting that points P8 and P9, associated with a low slug pattern frequency, result in capturing fewer unit cells. On the other hand, points P19 and P22, with higher frequencies, lead to capturing a greater number of unit cells.

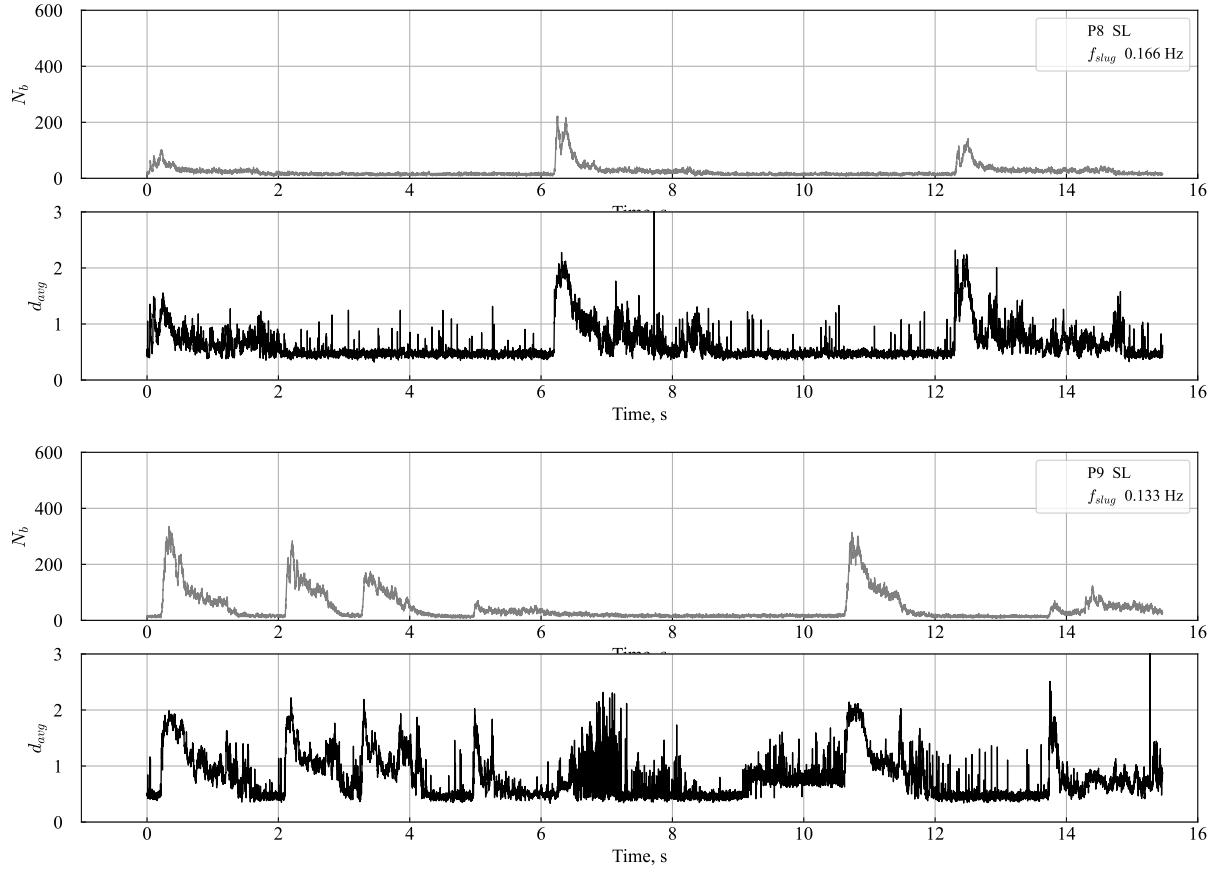


Figure B.4 – Series of bubble numbers N_b (gray line) and average bubble diameter d_{avg} (black line) for experimental point P8 and P9.

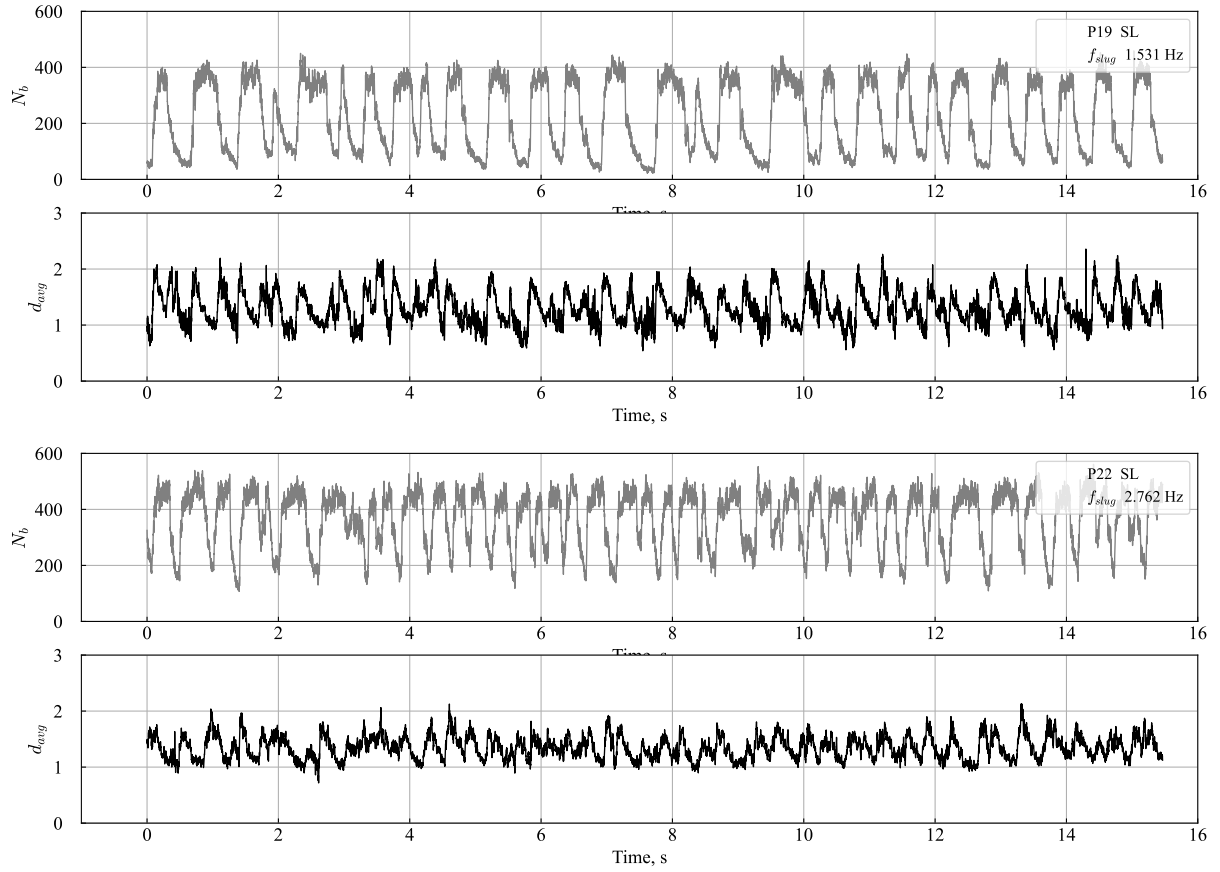


Figura B.5 – Series of bubble numbers N_b (gray line) and average bubble diameter d_{avg} (black line) for experimental point P19 and P22.

B.3 Conclusion

In summary, the SAM proves to be a comprehensive and flexible solution for image segmentation tasks, standing out for its adaptability to various demands, from edge detection to semantic segmentation. Its prompt-based training approach and the use of an extensive database of segmentation masks give it remarkable robustness. In the specific context of bubble segmentation in two-phase flows, SAM demonstrated effectiveness in generating masks automatically, eliminating the need for pre-training or manual annotation. The automatic mask generation methodology, coupled with the IoU technique for overlap adjustment, allowed for the precise identification of bubbles in images captured during gas-liquid flow experiments.